Investigation of A Fuzzy Integral Equation by Fuzzy Integral Transforms

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Issue: *Special Issue on Mathematical Computation in Combinatorics and Graph Theory in Mathematical Statistician and Engineering Applications*

Article Info
Page Number: 73 - 80
Publication Issue:
Vol 71 No. 3s3 (2022)

Article Received: *30 April 2022*
Revised: *22 May 2022*
Accepted: *25 June 2022*
Publication: *02 August 2022*

Abstract
In this paper, three different fuzzy integral transforms are used to solve fuzzy convolution Volterra integral equation of the second kind. These fuzzy transform methods reduc the problem to algebraic problem. Integral equations, some definitions and theorems about fuzzy integral transforms, and the convolution theorem are discussed in some details. Finally, illustrative realistic problem with convolution kernel represented by the gas diffusion phenomena is given to show the ability of the proposed methods.

Keywords: - integral equations, fuzzy transforms, Laplace transform, Tarig transform, Sadik transform, convolution theorem, gas diffusion.

2010 Mathematics Subject Classification: Primary 26A33, 44A05, Secondary 34A08, 34A07.

1. Introduction

Differential equations and integral equations are closely related because some issues can be expressed in either method. Integral equations have a significant role in both pure and applied mathematics, with numerous applications, especially in physical problems. The field of fuzzy integral equations is one of the more rapidly increasing topics, particularly in relation to fuzzy control, which has recently been established. The majority of mathematical models used in many different real-life problems are based on integral equations and fuzzy integral equations solved with fuzzy integral transforms such as Laplace, Tarig, Sadik, Sumudu, and Elzaki transforms (see papers [1-8,19-26]).
In this work, a fuzzy novel is introduced via the concepts of integral equations and integral transforms in four sections. Furthermore, a realistic problem related with gas diffuses phenomenon is discussed as an application for the solution of the convolution Volterra integral equation of the second kind using three different kinds of fuzzy integral transforms (Laplace, Tarig and Sadik).

2. Integral Equations

An integral equation is an equation in which the unknown function $\mu(x)$ (to be determined) appears under the integral sign. A typical form of an integral equation in $\mu(x)$ is:

$$\mu(x) = \gamma(x) + \lambda \int_{l(x)}^{h(x)} M(x,t)\mu(t)dt \quad (1)$$

where $M(x,t)$ is called the kernel of the integral equation, the function $\gamma(x)$ is called the driving term, $h(x)$ and $l(x)$ are the limits of integration and both of them may be variables, constants or mixed [10-16]. It is easily observed that the unknown function $\mu(x)$ appears under the integral sign at stated above, and out of the integral sign in most other cases. Integral equations arise naturally in physics, chemistry, biology and engineering [2-6, 8-14]

More details about the sources and origins of integral equations, the relation between differential equations and integral equations as well as how the initial value problem be converted to the form of integral equations can be found [5-8]. Other problems whose direct representation in terms of differential equation and their auxiliary conditions may also be decreasing to integral equation [7-13].

2.1. Types of Integral Equations

According to limits of integrations, the driving terms and the kernel, Arfken [4-10] gave a simple classification for integral equations, as follows:

1. If the limits of integration are fixed, the equation is called Fredholm integral equation, whereas if one of these limits is variable, then the equation will be Volterra integral equation.
2. If the unknown function μ(x) appears only under the integral sign, we shall label the equation as the first kind. While if it appears both inside and outside the integral sign, the equation be labeled as the second kind[8-14].

Symbolically, Fredholm integral equation of the first kind and second kind respectively can be written as follows:[14-20]

$\gamma(x)=\lambda\int_a^b M\,(x,t)\mu(t)dt,$ (2)

$\mu(x)=\gamma(x)+\lambda\int_a^b M\,(x,t)\mu(t)dt$ (3)

where h(x)=b, l(x)=a are constants.

Volterra integral equation of the first, second kind will be written respectively as:

$\gamma(x)=\lambda\int_a^x M\,(x,t)\mu(t)dt,$ (4)

$\mu(x)=\gamma(x)+\lambda\int_a^x M\,(x,t)\mu(t)dt$ (5)

2.2. Classification of the Kernel of Integral Equations [8-12]

The role of the known function (the kernel) M(x,t) is quite significant both from the problem and its solution. Mainly,we shall come across with the following classification of the kernel of the integral equations:

1.The kernel is said to be symmetric kernel (complex symmetric or Hermition) if M(x,t) = $\overline{M}$(x,t) (where the bar represents the complex conjugate). Easily, a real kernel is symmetric if M(x,t) = M(t,x).[8]

2.The kernel M(x,t) is called separable or degenerate kernel which means that M has been expressed as the sum of a finite number of terms, each of this sum is the product of function of x only and t only, i.e.[10-12]

$M(x,t)=\sum_{i=1}^{n} f_i\,(x)g_i(t).$ (6)

Obviously, $f_i(x)$ and $g_i(t)$ are linearly independent (or else some of finite number of characteristic values).

3.The kernel of the form M(x,t) = M(x-t) is called a difference kernel since it depends solely on the difference (x-t).

3. Fuzzy Integral Transforms Concepts

Fuzzy integral transforms method solves fuzzy differential equations (FDEs) corresponding to the fuzzy initial and boundary value problems. In this way fuzzy transforms reduces the problem of a

FDEs to an algebraic problem. This switching of operations on transforms is called calculus. This kind of fuzzy integral transforms method is practically the most important operational method. In this section we introduced some basic definitions and theorems that related to the fuzzy integral transforms. For more information we refer [5-16]

Definition 1: [1-4] A fuzzy number (triangular) is a mapping $u: R \longrightarrow [0,1]$ which satisfies the following properties:
u(x) is an upper semi-continuous function,
u(x) = 0 outside some interval [p,q],
a,b are real numbers such that $p \le a \le b \le q$ and
u(x) is monotonic increasing function on [p,a],
u(x) is monotonic decreasing function on [b,q],
u(x) = 1 for all $x \in [a,b]$.

Definition 2: [12,16] A fuzzy number u in parametric form by an ordered pair of functions $(\underline{u}(\alpha),\overline{u}(\alpha)), 0 \le \alpha \le 1$,which is satisfy the following requirements:
1. $\underline{u}(\alpha)$ is a bounded left -continuous non decreasing function over [0,1],
2. $\overline{u}(\alpha)$ is bounded left -continuous non-increasing function over [0,1],
3. $\underline{u}(\alpha) \le \overline{u}(\alpha)$, $\alpha \in [0,1]$.

Theorem 1: [1] Let f(x) be a fuzzy -valued function on [a,∞) and it is represented by $(\underline{f}(x,\alpha),\overline{f}(x,\alpha))$ for any fixed
$\alpha \in [0,1]$. Assume that $\underline{f}(x,\alpha)$ and $\overline{f}(x,\alpha)$ are Riemann-integrable functions on [a,b] for every $b \ge a$, and assume there are two positive functions $\underline{K}(\alpha)$ and $\overline{K}(\alpha)$ such that: $\int_a^b |\underline{f}(x,\alpha)| dx \le \underline{K}(\alpha)$ and $\int_a^b |\overline{f}(x,\alpha)| dx \le \overline{K}(\alpha)$, for every $b \ge a$. Then f(x) is an improper fuzzy Riemann-integral function on [0,∞) and the improper fuzzy Riemann -integral is a fuzzy number. Furthermore, we have: $\int_a^\infty f(x)\, dx = (\int_a^\infty \underline{f}(x,\alpha)\, dx, \int_a^\infty \overline{f}(x,\alpha)\, dx)$.

Proposition 1: [19-21] If each of f(x) and η(x) are fuzzy Riemann -integrable functions on [a,∞), then $f(x) \oplus \eta(x)$ is a fuzzy Riemann-integrable on [0, ∞).Moreover:
$\int_a^\infty (f(x) \oplus \eta(x))\, dx = \int_a^\infty f(x)\, dx \oplus \int_a^\infty \eta(x)\, dx$. (7)
It is well -known that H-derivative (differentiability in the sense of Hukuhara) for fuzzy mapping was initially introduced by Puri and Ralescu in 1983 [10] and it is based on the H-difference of sets.

Definition 3:[14-18] Let a, b ∈ R_F where R_F denotes the class of fuzzy subsets of real axis. If there exists c ∈ R_F such that a = b + d, then c is called the H-differential of a, b,or H -difference and it is denoted by $a \ominus b$.
Note that: in this work, the sign $\ominus$ always meant the H -difference as well as $a \ominus b \neq a+(-b)$.

Definition 4:[1-4] Let ξ: (a,b) → E,where E indicates the set of all fuzzy numbers on R and $x_o \in (a,b)$ be a strongly generalized differential at x_o.If there exists an element $\xi'(x_o) \in E$, such that:

For all h > 0 sufficiently small, $\exists\, \xi\,(x_o + h)\, \Theta\, \xi\,(x_o), \exists\, \xi(x_o)\, \Theta\, \xi(x_o - h)$ and the limits, $\frac{\xi\,(x_o+h)\Theta\,\xi(x_o)}{h} = \frac{\xi(x_o)\,\Theta\,\xi(x_o-h)}{h} = \xi'(x_o)$, or

For all h > 0 sufficiently small, $\exists\, \xi(x_o)\, \Theta\, \xi(x_o + h), \exists\, \xi(x_o - h)\, \Theta\, \xi(x_o)$ and the limits, $\frac{\xi(x_o)\Theta\,\xi(x_o+h)}{-h} = \frac{\xi(x_o-h)\,\Theta\,\xi(x_o)}{-h} = \xi'(x_o)$, or

For all h > 0 sufficiently small, $\exists\, \xi(x_o + h)\, \Theta\, \xi(x_o), \exists\, \xi(x_o - h)\, \Theta\, \xi(x_o)$ and the limits, $\frac{\xi(x_o+h)\Theta\,\xi(x_o)}{h} = \frac{\xi(x_o-h)\Theta\,\xi(x_o)}{-h} = \xi'(x_o), or$

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An important branch in fuzzy mathematics is the topics of fuzzy integral equations. Zadeh [7] was firstly introduced the concepts of fuzzy numbers and arithmetic operations. Whereas, the integration of fuzzy functions was initially coined out by Dubois and Prade [9]. Many researchers are discussed fuzzy Volterra integral equations, begins in Kaleva [11], Mordeson and Newman [16], and Seikkala [17], such integral equations are applied in control mathematical models.

The most recent methods in handling problems modelled under fuzzy environment are fuzzy Laplace transform (FLT), fuzzy Sumudu transfom (FST), fuzzy Tarig transform (F$\tilde{T}$T) and more. These kinds of transforms have been used for solving various kinds of fuzzy differential and integral equations. This fall under the topic of operational calculus and under this topic.

3.1. Fuzzy Laplace Transform [18-22]

Let f(x) be a continuous fuzzy -value function, suppose that f(x) ⊙ e^(-vx) is an improper fuzzy Rimann -integrable on [0,∞), then $\int_0^\infty f(x) \odot e^{-vx}dx$ is called fuzzy Laplace transform and is denoted by:

$$L[f(x)] = \int_0^\infty f(x) \odot e^{-vx}\, dx$$
$$= \left(\int_0^\infty \underline{f}(x,r)\, e^{-vx}dx, \int_0^\infty \bar{f}(x,r)\, e^{-vx}dx\right),$$

such that: $L[\underline{f}(x,r)]=\int_0^\infty \underline{f}(x,r)\, e^{-vx}dx$ and $L[\bar{f}(x,r)] = \int_0^\infty \bar{f}\,(x,r)e^{-vx}dx$,then
$L[f(x)]= (L[\underline{f}(x,r)], L[\bar{f}(x,r)]$.

3.2. Fuzzy Tarig Transform ($\tilde{T}$ -Transform) [1]

Let f (x) be a continuous fuzzy -valued function. Suppose that $\frac{1}{u}\int_0^\infty e^{\frac{-t}{u^2}} f(x)dx$ is an improper fuzzy Rimann -integrable on [0,∞),then $\frac{1}{u}\int_0^\infty e^{\frac{-t}{u^2}} f(x)dx$ is called $\tilde{T}$ -transform and it is denoted by:

$$\tilde{T}\,[f(x)] = \frac{1}{u}\int_0^\infty e^{\frac{-t}{u^2}}\, f(x)dx = \left(\frac{1}{u}\int_0^\infty e^{\frac{-t}{u^2}}\, \underline{f}(x)dx, \frac{1}{u}\int_0^\infty e^{\frac{-t}{u^2}}\, \bar{f}(x)dx\right), (x > 0)$$

using the definition of classical T-Transform:

$T(\underline{f}(x,r)= \frac{1}{u}\int_0^\infty e^{\frac{-t}{u^2}}\, \underline{f}(x)dx$ and $T\,(\bar{f}\,(x,r) = \frac{1}{u}\int_0^\infty e^{\frac{-t}{u^2}}\, \bar{f}(x)dx$.

Therefore: $\tilde{T}\,[f(x,r)]$ =(T[$\underline{f}(x,r)$],T [$\bar{f}(x,r)$]).

3.3 Fuzzy Sadik Transform: [14]

Let f (x) be a continuous fuzzy -valued function. Suppose that $\frac{1}{v^{\beta}}\int_0^{\infty} e^{-xv^{\delta}} f(x)\,dx$, (v is a complex variable, δ, β are any non zero real numbers) is an improper fuzzy Rimann -integrable on [0,∞),then $\frac{1}{v^{\beta}}\int_0^{\infty} e^{-xv^{\delta}} f(x)\,dx$ is called fuzzy Sadik transform (FST) and it is denoted by:

$S[f(x)] = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-xv^{\delta}} f(x)dx = (\frac{1}{v^{\beta}}\int_0^{\infty} e^{-xv^{\delta}} \underline{f}(x)dx, \frac{1}{v^{\beta}}\int_0^{\infty} e^{-xv^{\delta}} \bar{f}(x)dx)$

Using the definition of Sadik transform:

$s[\underline{f}(x,r)] = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-xv^{\delta}} \underline{f}(x,r)dx$, $s[\bar{f}(x,r) = \frac{1}{v^{\beta}}\int_0^{\infty} e^{-xv^{\delta}} \bar{f}(x,r)dx$.

Therefore,

$S[f(x,r)] = s[\underline{f}(x,r)], s[\bar{f}(f(x,r)]$.

4. Fuzzy Convolution Volterra Integral Equation of the Second Kind

Volterra integral equations appear when we convert initial value problem to an integral equation. In fact, the solution of Volterra integral equation is much easier than the solution of the original initial value problem [15]. Many problems of science and engineering like neutron diffusion problem, heat transfer problem, radiation transfer problem, electric circuit problem, etc, can be represent mathematically in terms of Volterra integral equation. As well, a topic of fuzzy integral equations that has attracted a growing interest for some time, has been developed, in particular in relation to fuzzy control[13].

In this section, we investigate the solution of fuzzy convolution Volterra integral equation (FCVI) of the second kind of the form:

$\mu(x) = \gamma(x) + \int_0^x M(x-t)\mu(t)dt, t \in [0,\infty)$ (8)

where M(x-t) is an arbitrary given fuzzy-valued convolution kernel function and μ(x) is a continuous fuzzy-valued function.

4.1. Fuzzy Convolution Theorem [15]

The convolution of two fuzzy -valued functions f (x) and η(x) are defined for x > 0 by:

$$(f * \eta)(x) = \int_0^x f(\tau)\eta(x-\tau)d\tau \quad (9)$$

which of course exists if f (x) and η(x) are piecewise continuous functions. Substituting u=x-τ gives: $(f * \eta)(x) = \int_0^x \eta(u) f(x-u)du = (\eta * f)(x)$. That is, the fuzzy convolution is commutative and the basic properties of the fuzzy convolution are:

$$c(f * \eta) = cf * \eta = f * c\eta,$$
$$f * (\eta * \omega) = (f * \eta) * \omega.$$

The following is an application for such kind of integral equation of the second kind.

4.2. Gas Diffuses Phenomenon

In many fields, including physics, chemistry and life sciences, the concept of diffusion appears widely, so in general we mean diffusion is the net movement of any thing (atom ions and molecules) from a region of higher concentration to a region of lower concentration [2-6]. In this example, we will assume that the diffusion of gas into the liquid is in along and narrow tube and that this process will take along period of time as the concentration of gas in the distance x from some initial point 0 (and is independent of time). So we will write the equation as follows:

μ (x) = D + A $\int_0^x M\ (x,r)\ \mu\ (t)dt$; (where D is constant).

Now, we will solve this equation with some fuzzy integral transforms as shown below:

A: L -Transform:

μ (x, r) = γ (x, r) + A $\int_0^x (\ x - t\ )\ \mu\ (t)dt$

μ(x, r) = (r, 2-r) γ (x, r) + A $\int_0^x (\ x - t\ )\ \ \mu\ (t\ )dt$

$\underline{\mu}$(x, r) = r γ (x, r) + A $\int_0^x (\ x - t\ )\ \ \underline{\mu}(t)$ dt, and

$\bar{\mu}$(x, r) = (2-r) γ (x, r) + A $\int_0^x (\ x - t\ )\ \ \underline{\mu}(t)dt$,

So we obtain L {$\underline{\mu}$(x, r)} and L {$\bar{\mu}$(x, r)} as follows

L {$\underline{\mu}$(x, r)} = L {r} + A L {$\int_0^x (\ x - t\ )\ \ \underline{\mu}(t)dt$}

L {$\underline{\mu}$(x, r)} = $\frac{r}{v} + A\ \cdot\ \frac{1}{v^2}$ L {$\underline{\mu}(x, r)$}

L {$\underline{\mu}$(x, r)} [1-$\frac{A}{v^2}$] = $\frac{r}{v}$

L {$\underline{\mu}$(x, r)} = (r) $\frac{v}{v^2 - A}$.

Now applying the inverse of Laplace transform on last equation, to get:

$\underline{\mu}$(x, r) = r cosh $\sqrt{A}$ x and $\bar{\mu}$(x, r) = (2-r) $\sqrt{A}$ x which can be written as:

μ (x, r) = (r, 2-r) cosh $\sqrt{A}$ x.

B: $\tilde{T}$ -Transform

$$T\ \{\underline{\mu}\ (x, r)\}\ =\ T\{r\}\ +\ A\ v\ [T\{x\}]\ \cdot\ T\ \{\underline{\mu}\ (x, r)\}$$

$$T\ \{\underline{\mu}\ (x, r)\}\ =\ r\ v\ +\ A\ v\ \cdot\ v^3\ \cdot\ T\ \{\underline{\mu}\ (x, r)\}$$

$$T\ \{\underline{\mu}\ (x, r)\}\ [1 - A\ v^4]\ =\ r\ v$$

$$T\ \{\underline{\mu}\ (x, r)\}\ =\ \frac{r\ v}{1 - A\ v^4}$$

$$\underline{\mu}\ (x, r)\ =\ r\ T^{-1}\ [\frac{v}{1 - A\ v^4}]\ =\ r\ cosh\ \sqrt{A}\ x\ and\ \bar{\mu}\ (x, r)\ =\ (2 - r)\ cosh\ \sqrt{A}\ x.$$

C: S -Transform

$$S\ \{\mu\ (x, r)\}\ =\ S\{(r, 2 - r)\ \gamma\ (x, r)\ +\ A\ S\{\int_0^\infty (\ x - t\ )\ \mu\ (t)dt\}$$

$$S\ \{\underline{\mu}\ (x, r)\}\ =\ S\ \{r\}\ +\ A\ V^\beta\ S\ \{x\}\ \cdot\ S\ \{(\underline{\mu}\ (x, r)\}$$

$$S\ \{\underline{\mu}\ (x, r)\}\ =\ \frac{r}{V^{\delta+\beta}} + AV^\beta\ \frac{1}{V^{2\delta+\beta}}\ \cdot S\ \{\underline{\mu}(x, r\ )\}$$

$$S\ \{\underline{\mu}\ (x, r)\ [1 - \frac{A\ V^\beta}{V^{2\delta+\beta}}] = \frac{r}{V^{\delta+\beta}}$$

$$S\ \{\underline{\mu}\ (x, r)\}\ =\ \frac{r\ V^\delta}{V^\beta\ (V^{2\delta} - A\ )}$$

$$\underline{\mu}\ (x, r)\ =\ r\ S^{-1}\ [\frac{V^\delta}{V^\beta\ (V^{2\delta} - A)}] = r\ cosh\ \ \sqrt{A}\ x\ and\ \bar{\mu}\ (x, r)\ =\ (2 - r)\ cosh\ \ \sqrt{A}\ x.$$

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