Intuitionistic Fuzzy RG-ideals of RG-algebra

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Abstract

The purpose of this paper is to introduce the concept of Intuitionistic fuzzy of RG-algebra, as well as to state and prove various theorems and properties. Intuitionistic fuzzy RG-algebras and Intuitionistic fuzzy RG-ideals are also investigated for their fuzzy relations.

Keywords: RG-algebra, intuitionistic fuzzy RG-subalgebra, intuitionistic fuzzy RG-ideal, the homeomorphic of them.

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1. Introduction

The concept of fuzzy sets was first introduced by Zadeh [16] and has since undergone a number of expansions.One such these is Atanassov's [1-4] idea of intuitionistic fuzzy sets.
Intuitionistic fuzzy sets give both degrees of membership and non-membership of an element in a given set, while fuzzy sets give only a degree of membership.Both degrees fall inside the range [0,1], hence the sum should not be greater than 1.The class BCK-algebra is a legitimate subclass of the class BCI-algebras, as is well understood.Intuitionistic fuzzy H-ideals in BCK algebras were recently proposed by Senapati and coworkers [14-15, 17-20].
There has been a lot of discussion on fuzzy translations and ideals in BCK/BCI-algebras.
On the one hand, R.K. Omar [12] proposed the idea of RGO algebras, RG-ideals, and RG subalgebras and examined their connections, while P. Patthanangkoor [13] developed the idea of RG algebra homomorphism and looked into certain associated characteristics.
According to Hameed and colleagues, fuzzy subalgebras of RG-algebras as well as fuzzy ideals of RG-algebras are new ideas that have been examined in [9].According to [10]. T. Hameed and S.M. Abrahem proposed the concept of doubt fuzzy RG-ideals of RG-algebras and investigated the homomorphism image and inverse image of doubt fuzzy RG-ideals.
RG-subalgebras and RG-ideals on RG-algebras are introduced, and a wide range of their properties are examined, in this study. There are also connections between intuitionistic fuzzy RG-algebras.

2. Preliminaries

Now, we give some definitions and preliminary results needed in the later sections.

Definition 2.1. [12]: An algebra $(X;*,0)$ is called RG-algebra if the following axioms are satisfied: $\forall \rho, \tau, z \in X$,

(i) $\rho * 0 = \rho$,

(ii) $\rho * \tau = (\rho * z) * (\tau * z)$,

(iii)$\rho * \tau = \tau * \rho = 0$ imply $\rho = \tau$.

Remark 2.2. [12]: In $(X;*,0)$ an RG-algebra, we define a binary relation $(\leq)$ by putting $\rho \leq \tau$ if and only if $\rho * \tau = 0$.

Definition 2.3. [12,13]: Let $(X;*,0)$ be an RG-algebra, a nonempty subset Iof X is called an RG-ideal of X if $\forall \rho, \tau \in X$

i) $0 \in I$,

ii) $\rho * \tau \in I$ and $0 * \rho \in I$ imply $0 * \tau \in I$.

Proposition 2.4. [12,13]: In an RG-algebra $(X;*,0)$, every RG-ideal is a subalgebra of X.

Proposition 2.5. [12]: In any RG-algebra $(X;*,0)$, the following hold: $\forall \rho, \tau, z \in X$,

i) $\rho * \rho = 0$,

ii) $0 * (0 * \rho) = \rho$,

iii) $\rho * (\rho * \tau) = \tau$,

iv) $\rho * \tau = 0$ if and only if $\tau * \rho = 0$,

v) $\rho * 0 = 0$ implies $\rho = 0$,

vi) $0 * (\tau * \rho) = \rho * \tau$.

Proposition 2.6. [12]: In any RG-algebra $(X;*,0)$, the following hold: $\forall \rho, \tau, z \in X$,

i) $(\rho * \tau) * (0 * \tau) = \big(\rho * (0 * \tau)\big) * \tau = \rho$,

ii) $\rho * (\rho * (\rho * \tau)) = \rho * \tau$,

iii) $(\rho * \tau) * z = (\rho * z) * \tau$,

iv) $\rho * \tau = (z * \tau) * (z * \rho)$,

v) $\big((\rho * \tau) * (\rho * z)\big) * (z * \tau) = 0$.

Theorem 2.7. [13]: If $f: (X; *, 0) \to (Y; *`, 0`)$ is a homomorphism of an RG-algebras

respectively X, Y, then

1) $f(0) = 0'$.

2) f is injective if and only if ker f = {0}.

Definition 2.8. [16]: Let$(X;*,0)$ be a nonempty set, a fuzzy subset μ of X is a function $\mu: X \to [0,1]$.

Definition 2.9. [16]: For any t∈ [0,1] and a fuzzy subset μ of a nonempty set X, the set

$U(\mu, t) = \{\rho \in X \mid \mu(\rho) \geq t\}$ is called an upper level cut of μ, and the set

$L(\mu, t) = \{\rho \in X \mid \mu(\rho) \leq t\}$ is called a lower level cut of μ.

Definition 2.10.[9]: Let $(X;*,0)$ be an RG-algebra and S be a nonempty subset of X.Then S is called an RG-subalgebra of X if $\rho * \tau \in S$, for any $\rho, \tau \in S$.

Proposition 2.11. [9]: In an RG-algebra $(X;*,0)$ every RG-ideal is a RG-subalgebra of X.

Definition 2.12.[9]: Let $(X;*,0)$ be an RG-algebra, a fuzzy subset μ of X is called a fuzzy RG-subalgebra of X, if $\forall \rho, \tau \in X$, $\mu(\rho * \tau) \geq \min\{\mu(\rho), \mu(\tau)\}$ sets.

Definition 2.13.[9]: Let $(X;*,0)$ be an RG-algebra, a fuzzy subset μ of X is called a fuzzy RG-ideal of X if it satisfies the following conditions: $\forall x, y \in X$,

(i) $\mu(0) \geq \mu(\rho)$,

(v) $\mu(0 * \tau) \geq \min\{\mu(\rho * \tau), \mu(0 * \rho)\}$.

Proposition 2.14. [9]: Every fuzzy RG-ideal of RG-algebra $(X;*,0)$ is a fuzzy RG-subalgebra of X.

Proposition 2.15.[9]: 1- The intersection of any set of fuzzy RG-subalgebras of RG-algebra $(X;*,0)$ is also fuzzy RG-subalgebra of X.

2- The union of any set of fuzzy RG-subalgebras of RG-algebra is also fuzzy RG-subalgebra, where is chain (Noetherian).

3- The intersection of any set of fuzzy RG-ideals of RG-algebra $(X;*,0)$ is also fuzzy RG-ideal of X.

4- The union of any set of fuzzy RG-ideals of RG-algebra is also fuzzy RG-ideal, where is chain (Noetherian).

Definition 2.16.[1,2]: Let f: $(X;*,0)\to(Y;*',0')$ be a homeomorphism from the set X into the set Y. If μ is a fuzzy subset of X, then the fuzzy subset f(μ) in Y defined by:

$$f(\mu)(\tau)=\begin{cases}\{\sup\{\mu(\rho): \rho \in f^{-1}(\tau)\} & \text{if } f^{-1}(\tau) = \{\rho \in X, f(\rho) = \tau\} \neq \emptyset \\ 0 & \text{otherwies}\end{cases}$$

is said to be the image of μ under f.

Similarly, if β is a fuzzy subset of Y, then the fuzzy subset μ =(β ∘ f) in X, (i.e. the fuzzy subset defined by μ (ρ)=β(f (ρ)), for all ρ ∈X) is called the pre-image of β under f.

Definition 2.17.[8]:

1) fuzzy subset μ of algebra $(X; *, 0)$ has inf property if for any subset T of X, there exist t0∈ T such that μ (t0)=$\inf_{t\in T}$ μ (t).

2) fuzzy subset μ of algebra $(X; *, 0)$ has inf property if for any subset T of X, there exist t0∈ T such that $\mu(t_0) = \sup$ {μ(t)| t∈T}.

Remark 2.18.[1,2]: fuzzy subset ℵ in X is defined as $\aleph = \{(\rho, \mu_\aleph(\rho))|\ \rho \in X\}$ where $\mu_\aleph(\rho)$ denotesto the degree of the membership value of ρ in ℵ and $0 \le \mu_\aleph(\rho) \le 1$.

Definition 2.19. [1]: An intuitionistic fuzzy subset ℵ in a nonempty set X is an object having the form $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) \mid \rho \in X\}$or ℵ =$(\mu_\aleph, \nu_\aleph)$, where the functions $\mu_\aleph$: X → [0,1] and $\nu_\aleph$: X → [0,1] denotes to the degree of the membership and the degree of non membership respectively, and $0 \le \mu_\aleph(\rho), \nu_\aleph(\rho) \le 1$, for all ρ∈X.

Definition 2.20.[10]: Let (X; *,0) be an RG-algebra. μ be a fuzzy subset of X, μ is called doubt fuzzy RG-subalgebra of X if for all $\rho, \tau \in X$ $\mu(\rho * \tau) \le \max\{\mu(\rho), \mu(\tau)\}$,

Definition 2.21.[10]: Let (X;∗ ,0) be an RG-algebra, a fuzzy subset μ of X is called a doubt fuzzy RG-ideal of X if it satisfies the following conditions: $\forall x, y \in X$,

1. $\mu(0) \le \mu(\rho)$.

2. $\mu(0 * \tau) \le \max\{\mu(\rho * \tau), \mu(0 * \rho)\}$.

Proposition 2.22.[10]: Every doubt fuzzy RG-ideal of RG-algebra $(X; *, 0)$ is a doubt fuzzy RG-subalgebra of X.

Definition 2.23.[1]: If $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho))|\ \rho \in X\}$ and $B = \{(\rho, \mu_B(\rho), \nu_B(\rho))|\rho \in X\}$ are two intuitionistic fuzzy subsets of X, then

1)$A \subseteq B$ if and only if $x \in X$, $\mu_A(x) \le \mu_B(x)$ and$v_A(x) \ge v_B(x)$.

2)$A = B$ if and only if $x \in X$, $\mu_A(x) = \mu_B(x)$and $v_A(x) = v_B(x)$.

3)$A \cap B = \{(\rho, (\mu_A \cap \mu_B)(x), (v_A \cup v_B)(x)\ |x \in X\}$.

4)$A \cup B = \{(\rho, (\mu_A \cup \mu_B)(x), (v_A \cap v_B)(x)\ |x \in X\}$.

Proposition 2.24.[10]:

1- The intersection of any set of doubt fuzzy RG-subalgebras of RG-algebra $(X; *, 0)$ is also doubt fuzzy RG-subalgebra of X, where is chain (Arterian).

2- The union of any set of doubt fuzzy RG-subalgebras of RG-algebra is also doubt fuzzy RG-subalgebra.

3- The intersection of any set of doubt fuzzy RG-ideals of RG-algebra $(X; *, 0)$ is also doubt fuzzy RG-ideal of X, where is chain (Arterian).

4- The union of any set of doubt fuzzy RG-ideals of RG-algebra is also doubt fuzzy RG-ideal.

Definition 2.25. [1]: ℵ mapping f: (X;∗ ,0) → (Y;∗ ,0)be a homeomorphism of BCK-algebra for any

IFS $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) | \rho \in X\}$ in Y, we define

IFS $\aleph^f = \left\{\left(\rho, \mu_\aleph^f(\rho), \nu_\aleph^f(\rho)\right) \middle| \rho \in X\right\}$ in X by $\mu_\aleph^f(\rho) = \mu_A(f(\rho))$,

$\nu_\aleph^f(\rho) = \nu_A(f(\rho)), \forall \rho \in X$.

3. Intuitionistic Fuzzy RG-subalgebras of RG-algebra

In this section, we give the concept of an intuitionistic fuzzy RG-subalgebras of RG-algebra X.

Definition 3.1. Let $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) | \rho \in X\}$ be an intuitionistic fuzzy subset of

RG-algebra (X; *,0). ℵ is said to be an intuitionistic fuzzy RG-subalgebra of X if

1) $\mu_\aleph(\rho * \tau) \geq \min\{\mu_\aleph(\rho), \mu_\aleph(\tau)\}$,

2) $\nu_\aleph(\rho * \tau) \leq \max\{\nu_\aleph(\rho), \nu_\aleph(\tau)\}$.

That mean $\mu_\aleph$ is a fuzzy RG-subalgebra and $\nu_\aleph$ is a doubt fuzzy RG-subalgebra.

Example 3.2. Let X={0, 1, 2, 3} in which ∗ is defined by the following table:

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $\mu(x) = \begin{cases} 0.8 & x = 0 \\ 0.3 & x \in \{1,2,3\} \end{cases}$, $\nu(x) = \begin{cases} 0.2 & x \in \{0,1\} \\ 0.4 & x \in \{2,3\} \end{cases}$ $\mu_\aleph$ is a fuzzy RG-subalgebra and $\nu_\aleph$ is a doubt fuzzy RG-subalgebra, then $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) | \rho \in X\}$ is intuitionistic fuzzy RG-subalgebra.

Proposition 3.3. Every intuitionistic fuzzy RG-subalgebra $\{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) | \rho \in X\}$ of RG-algebra (X; *,0), satisfies the inequalities $\mu_\aleph(0) \geq \mu_\aleph(\rho)$ and $\nu_\aleph(0) \leq \nu_\aleph(\rho)$, for all $\rho \in X$.

Proof. For any $\rho \in X$, we have $\mu_\aleph(0) = \mu_\aleph(\rho * \rho) \geq \min\{\mu_\aleph(\rho), \mu_\aleph(\rho)\} = \mu_\aleph(\rho)$

and $\nu_\aleph(0) = \nu_\aleph(\rho * \rho) \leq \max\{\nu_\aleph(\rho), \nu_\aleph(\rho)\} = \nu_\aleph(\rho)$. ■

Proposition 3.4: An intuitionistic fuzzy subset $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) | \rho \in X\}$ is an intuitionistic fuzzy RG- subalgebra of RG-algebra (X; *,0), if for any $t \in [0,1]$, the set $U(\mu_\aleph, t)$ and $L(\nu_\aleph, s)$ are RG-subalgebras.

Proof. Let $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG-subalgebra of X and set $U(\mu_{\aleph}, t) \neq \emptyset \neq L(\nu_{\aleph}, s)$.

If follows that for $\rho \in U(\mu_{\aleph}, t)$, $\tau \in U(\mu_{\aleph}, t)$,then $\mu_{\aleph}(\rho) \geq t, \mu_{\aleph}(\tau) \geq t$ which follow $\mu_{\aleph}(\rho * \tau) \geq \min\{\mu_{\aleph}(\rho), \mu_{\aleph}(\tau)\} \geq t$, So that $\rho * \tau \in U(\mu_{\aleph}, t)$.

Hence $U(\mu_{\aleph}, t)$ is an RG-subalgebra of X.

we prove that $L(\nu_{\aleph}, s)$ is an RG-subalgebra of X.

$\rho \in L(\nu_{\aleph}, s)$ and $\tau \in L(\nu_{\aleph}, s)$ and $\nu_{\aleph}(\rho) \leq s$ and $\nu_{\aleph}(\tau) \leq s$

If follows that $\nu_{\aleph}(\rho * \tau) \leq \max\{\nu_{\aleph}(\rho), \nu_{\aleph}(\tau)\} \leq s$, So that $\rho * \tau \in L(\nu_{\aleph}, t)$.

Hence $L(\nu_{\aleph}, t)$ is an RG-subalgebra of X.■

Proposition 3.5: In an intuitionistic fuzzy subalgebra $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$, if the upper level and lower level of (X; *,0), are RG-subalgebra, for all $t \in [0,1]$, then $\aleph$ is an intuitionistic fuzzy RG-subalgebra of X.

Proof. Assume that for each $t \in [0,1]$ the set $U(\mu_{\aleph}, t)$ and $L(\nu_{\aleph}, s)$ are RG-subalgebra of X. If there exist $\acute{\rho}, \acute{\tau} \in X$ be such that $\mu_{\aleph}(\acute{\rho} * \acute{\tau}) < \min\{\mu_{\aleph}(\acute{\rho}), \mu_{\aleph}(\acute{\tau})\}$, then $\acute{t} = \frac{1}{2}(\mu_{\aleph}(\acute{\rho} * \acute{\tau}) + \min\{\mu_{\aleph}(\acute{\rho}), \mu_{\aleph}(\acute{\tau})\})$

$\mu_{\aleph}(\acute{\rho} * \acute{\tau}) < \acute{t}, \acute{\rho} * \acute{\tau} \notin U(\mu_{\aleph}, \acute{t})$ is not RG-subalgebra that mean it is contradiction.

Now, $\nu_{\aleph}(\acute{\rho} * \acute{\tau}) > \max\{\nu_{\aleph}(\acute{\rho}), \nu_{\aleph}(\acute{\tau})\}$, then $\acute{s} = \frac{1}{2}(\nu_{\aleph}(\acute{\rho} * \acute{\tau}) + \max\{\nu_{\aleph}(\acute{\rho}), \nu_{\aleph}(\acute{\tau})\})$

$\nu_{\aleph}(\acute{\rho} * \acute{\tau}) < \acute{s}, \acute{\rho} * \acute{\tau} \notin L(\nu_{\aleph}, \acute{s})$ is not doubt RG- subalgebra that mean it is contradiction

Hence $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG-subalgebra X. ■

Remark 3.6. Let (X; *,0) be an RG-algebra.

1) If $\mu_{\aleph}$ is an RG-subalgebra of X, then $\bar{\mu}_{\aleph} = 1 - \mu_{\aleph}$ is a doubt fuzzy RG-subalgebra of X.

2) If $\nu_{\aleph}$ is a doubt fuzzy RG-subalgebra of X, then $\bar{\nu}_{\aleph} = 1 - \nu_{\aleph}$ is a fuzzy RG-subalgebra of X.

3) If $\mu_{\aleph}$ is an RG-ideal of RG-algebra of X, then $\bar{\mu}_{\aleph} = 1 - \mu_{\aleph}$ is a doubt fuzzy RG- ideal of X.

4) If $\nu_{\aleph}$ is a doubt fuzzy RG- ideal of X, then $\bar{\nu}_{\aleph} = 1 - \nu_{\aleph}$ is a fuzzy RG-ideal of X.

Theorem 3.7. Let $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy set of an RG-algebra (X; *,0). $\aleph$ is an intuitionistic fuzzy RG-subalgebra of X if and only if the fuzzy set $\mu_{\aleph}(\rho)$ is a fuzzy RG-subalgebra, $\nu_{\aleph}(\rho)$ is a doubt fuzzy RG-subalgebra of X.

Proof . Since $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG-subalgebra.

Cleary, $\mu_{\aleph}(\rho)$ is a fuzzy RG-subalgebra of X. For all $\rho, \tau \in X$, we have

$\bar{\nu}_{\aleph}(\rho * \tau) = 1 - \nu_{\aleph}(\rho * \tau) \geq 1 - \max\{\nu_{\aleph}(\rho), \nu_{\aleph}(\tau)\}$

$\geq \min\{1 - \nu_{\aleph}(\rho), 1 - \nu_{\aleph}(\tau) \geq \min\{\bar{\nu}_{\aleph}(\rho), \bar{\nu}_{\aleph}(\tau)\}$

Hence $\bar{\nu}_{\aleph}$is fuzzy RG-subalgebra of X.

The conversely, assume that $\mu_{\aleph}, \bar{\nu}_{\aleph}$ are fuzzy RG-subalgebra of X, for every $\rho, \tau \in X$,

we get $\mu_{\aleph}(\rho * \tau) \geq \min\{\mu_{\aleph}(\rho), \mu_{\aleph}(\tau)\}$ and

$1 - \nu_{\aleph}(\rho * \tau) = \bar{\nu}_{\aleph}(\rho * \tau) \geq \min\{\bar{\nu}_{\aleph}(\rho), \bar{\nu}_{\aleph}(\tau)\}$

$= \min\{1 - \nu_{\aleph}(\rho), 1 - \nu_{\aleph}(\tau)\}$

$= 1 - \max\{\nu_{\aleph}(\rho), \nu_{\aleph}(\tau)\}$

That is $\nu_{\aleph}(\rho * \tau) \leq \max\{\nu_{\aleph}(\rho), \nu_{\aleph}(\tau)$.

Hence $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ an intuitionistic fuzzy RG-subalgebra. ■

Theorem 3.8. Let f: $(X; *, 0) \to (Y; *`, 0`)$ be a homomorphism of an RG-algebras (X; *,0), (Y; *,0) respectively. If $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ is an intuitionistic fuzzy RG-subalgebra of Y, then an

$\aleph^{f} = \left\{\left(\rho, \mu_{\aleph}^{f}(\rho), \nu_{\aleph}^{f}(\rho)\right) \middle| \rho \in X\right\}$ is an intuitionistic fuzzy RG-subalgebra of X.

Proof . We first have that, $\forall \rho, \tau \in X$,then

$\min\{\mu_{\aleph}^{f}(\rho), \mu_{\aleph}^{f}(\tau)\} = \min\{\mu_{\aleph}(f(\rho)), \nu_{\aleph}(f(\tau)\} \leq \mu_{\aleph}(f(\rho * \tau)) = \mu_{\aleph}^{f}(\rho * \tau)$ and

$$\max\{\nu_{\aleph}^{f}(\rho), \nu_{\aleph}^{f}(\tau)\} = \max\{\nu_{\aleph}(f(\rho)), \nu_{\aleph}(f(\tau))\} \geq \nu_{\aleph}(f(\rho * \tau)) = \nu_{\aleph}^{f}(\rho * \tau).$$

$\aleph^{f} = \left\{\left(\rho, \mu_{\aleph}^{f}(\rho), \nu_{\aleph}^{f}(\rho)\right) \middle| \rho \in X\right\}$ is an intuitionistic fuzzy RG-subalgebra of X.■

Theorem 3.9. Let f: $(X; *, 0) \to (Y; *`, 0`)$ be a homomorphism of an RG-algebras algebras (X; *,0), (Y; *,0) respectively. If an $\aleph^{f} = \left\{\left(\rho, \mu_{\aleph}^{f}(\rho), \nu_{\aleph}^{f}(\rho)\right) \middle| \rho \in X\right\}$ is an intuitionistic fuzzy RG-subalgebra of X, then $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ is an intuitionistic fuzzy RG-subalgebra of Y.

Proof . We first have that, $\forall \rho, \tau \in Y$,then f (a)=ρ, f (b)=τ, and $f(a * b) = x *' y$, for some a, b∈ X, $\mu_{\aleph}^{f}(\rho) = \mu_{\aleph}(f(a))$, $\mu_{\aleph}^{f}(\tau) = \mu_{\aleph}(f(b))$, and $\mu_{\aleph}^{f}(\rho *' \tau) = \mu_{\aleph}(f(a * b))$,

$\mu_{\aleph}^{f}(\rho *' \tau) = \mu_{\aleph}(f(a * b))$

$\geq \min\{\mu_{\aleph}(f(a)), \mu_{\aleph}(f(b))\}$

$= \min\{\mu_{\aleph}^{f}(\rho), \mu_{\aleph}^{f}(\tau)\}$ and

, $\nu_{\aleph}^{f}(\rho) = \nu_{\aleph}(f(a))$, $\nu_{\aleph}^{f}(\tau) = \nu_{\aleph}(f(b))$ And

$\nu_{\aleph}^{f}(\rho *'\tau) = \nu_{\aleph}(f(a * b))$

$\leq \max\{\nu_{\aleph}(f(a)), \nu_{\aleph}(f(b))\}$
$=\max\{\nu_{\aleph}^{f}(\rho), \nu_{\aleph}^{f}(\tau)\}$

Hence $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) | \rho \in X\}$is an intuitionistic fuzzy RG-subalgebra of Y. ■

Definition 3.10: Let $\aleph_i = \{(\rho, \mu_{\aleph_i}(\rho), \nu_{\aleph_i}(\rho)) \mid \rho \in X\}$ be a an intuitionistic subset of RG-algebra (X;*,0) where $i \in \Lambda$, then

1.The R- intersection of any set of intuitionistic subsets of X is $(\cap \mu_{\aleph_i})(\rho) = \inf \mu_{\aleph_i}(\rho), (\cup \nu_{\aleph_i})(\rho) = \sup \nu_{\aleph_i}(\rho)$.

2.The P- intersection of any set of intuitionistic subsets of X is $(\cap \mu_{\aleph_i})(\rho) = \inf \mu_{\aleph_i}(\rho), (\cap \nu_{\aleph_i})(\rho) = \inf \nu_{\aleph_i}(\rho)$.

3.The P-union of any set of intuitionistic subsets of X is $(\cup \mu_{\aleph_i})(\rho) = \text{sub } \mu_{\aleph_i}(\rho), (\cup \nu_{\aleph_i})(\rho) = \sup \nu_{\aleph_i}(\rho)$.

4.The R-union of any set of intuitionistic subsets of X is $(\cup \mu_{\aleph_i})(\rho) = \text{sub } \mu_{\aleph_i}(\rho), (\cap \nu_{\aleph_i})(\rho) = \inf \nu_{\aleph_i}(\rho)$.

Theorem 3.11: Let $\{\aleph_i | i = 1,2,3, \dots\}$ be a family of intuitionistic fuzzy RG-subalgebra of RG-algebra (X;*,0), then the R- intersection of intuitionistic $\aleph_i$ is an intuitionistic fuzzy RG-subalgebra of X is intuitionistic fuzzy RG-subalgebra of X is an intuitionistic fuzzy RG-subalgebra where $\cap \aleph_i = (\inf \mu_{\aleph_i}(\rho), \sup \nu_{\aleph_i}(\rho))$.

Proof . Let $\rho, \tau \in \cap_{\aleph_i}$,then $\rho, \tau \in \aleph_i$ for all $i \in \Lambda$

$\cap \mu_{\aleph_i}(\rho * \tau) = \min\{\mu_{\aleph_i}(\rho*\tau)\} \geq \min\{\min\{\mu_{\aleph_i}(\rho), \mu_{\aleph_i}(\tau)\}\} \geq \min\{\cap \mu_{\aleph_i}(\rho), \cap \mu_{\aleph_i}(\tau)\}$ and

$\cup \nu_{\aleph_i}(\rho * \tau) = \max\{\nu_{\aleph_i}(\rho*\tau)\} \leq \max\{\max\{\nu_{\aleph_i}(\rho), \nu_{\aleph_i}(\tau)\}\} \leq \max\{\cup \nu_{\aleph_i}(\rho), \cup \nu_{\aleph_i}(\tau)\}$.

Then R-intersection of intuitionistic $\aleph_i$ is an intuitionistic fuzzy RG-subalgebra of X.■

Proposition 3.12: The P-intersection of any set $\aleph_i$ of intuitionistic subset of X, then the P-intersection of $\aleph_i$ is an intuitionistic fuzzy RG-subalgebra of X, where $\nu_{\aleph_i}$ chain (Arterian).

Proof . By using Proposition (2.25) and Proposition (2.22).■

Remark 3.14. If $\nu_{\aleph_i}$ is not chain in Proposition (3.13), then it is not true as the following example.

Example 3.15. Consider X in Example (3.2)

X	0	1	2	3
$\mu_ℵ$	0.9	0.9	0.2	0.2
μ_B	0.7	0.1	0.7	0.1
$\mu_ℵ \cup \mu_B$	0.9	0.9	0.7	0.2
$\mu_ℵ \cap \mu_B$	0.7	0.1	0.2	0.1
$v_ℵ$	0.2	0.7	0.2	0.7
v_B	0.3	0.8	0.9	0.3
$v_ℵ \cup v_B$	0.3	0.8	0.9	0.7
$v_ℵ \cap v_B$	0.2	0.7	0.2	0.3

It is easy to show that inf $\mu_{ℵ_i}(\rho)$ is a fuzzy RG-subalgebra, but $v_ℵ \cap v_B$ is not doubt fuzzy RG-subalgebra of X, since $\rho = 2, \tau = 3$,

$v_ℵ \cap v_B\ (2 * 3) = 0.7\ \nleq\ \max\{v_ℵ \cap v_B\ (2), v_ℵ \cap v_B\ (3)\} = 0.3$

Proposition 3.15. The P-union of any set $ℵ_i$ of intuitionistic subset of X, then the P-union $ℵ_i$ is an intuitionistic fuzzy RG-subalgebra of X, where $\mu_{ℵ_i}$ be a chain (Notherian)

Proof . By using Proposition (2.26) and Proposition (2.23).■

Remark 3.16. If $\mu_{ℵ_i}$ is not chain in Proposition (3.15), then it is not true as the following example.

Example 3.18. In the Example (3.14), then $(\mu_ℵ \cup \mu_B)$ are not fuzzy RG-subalgebra of X since

$(\mu_ℵ \cup \mu_B)(0 * 3) = (\mu_ℵ \cup \mu_B)(3) = 0.2 \ngeq 0.7$=min $\{(\mu_ℵ \cup \mu_B)(1 * 3), (\mu_ℵ \cup \mu_B)(0 * 1)\}$ $(\mu_ℵ \cup \mu_B)(3) = 0.2 \ngeq$ 0.7=min $\{(\mu_ℵ \cup \mu_B)(2), (\mu_ℵ \cup \mu_B)(1)\}$.

Proposition 3.18: The R-union of any set $ℵ_i$ of intuitionistic subset of an RG-algebra (X; *,0), then the R-union $ℵ_i$ is an intuitionistic fuzzy RG-subalgebra of X, where $v_{ℵ_i}$ is chain (Arterian) and $\mu_{ℵ_i}$, chain (Notherian).

Proof . By using Proposition (2.15(2)) and Proposition (2.24(2)). ■

Remark 3.19. If $\mu_{ℵ_i}$ is not chain in Proposition (3.18), as seen in the following example, this is not the case.

Example 3.20. Consider X in Example (3.2)

X	0	1	2	3
$\mu_ℵ$	0.9	0.9	0.2	0.2
μ_B	0.7	0.1	0.7	0.1
$\mu_ℵ \cup \mu_B$	0.9	0.9	0.7	0.2
$v_ℵ$	0.2	0.2	0.4	0.4
v_B	0.3	0.5	0.3	0.5
$v_ℵ \cap v_B$	0.2	0.2	0.3	0.4

Then $(\mu_ℵ \cup \mu_B)$ are not fuzzy RG-subalgebra of X since if ρ=2, τ=1
$\mu_ℵ \cup \mu_B(2 * 1) = \mu_ℵ \cup \mu_B(3) = 0.2 \ngeq$ 0.7=min $\{(\mu_ℵ \cup \mu_B)(2), (\mu_ℵ \cup \mu_B)(1)\}$

$\mu_ℵ \cup \mu_B(3) = 0.2 \ngeq$ 0.7=min{0.7,0.9}.

4. Intuitionistic Fuzzy RG-ideals of RG-algebra

In this section, we give the concept of an intuitionistic fuzzy RG-ideals of RG-algebra X.

Definition 4.1. Let $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy subset of RG-algebra (X; *,0). ℵ is said to be an intuitionistic fuzzy RG-ideal of X if, for all ρ, τ ∈X, then

1) $\mu_{\aleph}(0) \geq \mu_{\aleph}(\rho)$ and $\nu_{\aleph}(0) \leq \nu_{\aleph}(\rho)$,

2) $\mu_{\aleph}(0 * \tau) \geq \min\{\mu_{\aleph}(\rho * \tau), \mu_{\aleph}(0 * \rho)\}$ and $\nu_{\aleph}(0 * \tau) \leq \max\{\nu_{\aleph}(\rho * \tau), \nu_{\aleph}(0 * \rho)\}$.

That means $\mu_{\aleph}$ is a fuzzy RG-ideal and $\nu_{\aleph}$ is a doubt fuzzy RG-ideal.

Example 4.2. Let X ={a,b,c}with ∗ and constand (0) is defined by:

*	0	a	b	c
0	0	c	b	a
a	a	b	c	0
b	b	a	0	c
c	c	0	a	b

$\mu_A(x) = \begin{cases} 0.3 & x = 0 \\ 0.1 & x \in \{a, b, c\} \end{cases}$ and $\nu_A(x) = \begin{cases} 0.2 & x = 0 \\ 0.3 & x \in \{a, b, c\} \end{cases}$. We can see that $\mu_{\aleph}$ is a fuzzy RG-ideal and $\nu_{\aleph}$ is doubt-fuzzy RG-ideal of X, then ℵ is an intuitionistic fuzzy RG-ideal of X.

Proposition 4.3. If an intuitionistic fuzzy subset $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ is an intuitionistic fuzzy RG- ideal of RG-algebra (X; *,0), then for any t ∈[0,1], the set $U(\mu_{\aleph}, t)$ and $L(\nu_{\aleph}, s)$ are RG-ideals of X.

Proof. Let $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG- ideal of X and set $U(\mu_{\aleph}, t) \neq \emptyset \neq L(\nu_{\aleph}, s)$. Since $\mu_{\aleph}(0) \geq t, \nu_{\aleph}(0) \leq s$, let $\rho, \tau \in X$ be such that $\rho * \tau \in U(\mu_{\aleph}, t)$ and $0 * \rho \in U(\mu\_\aleph, t)$ and $\mu_{\aleph}(\rho * \tau) \geq t$ and $\mu_{\aleph}(0 * \rho) \geq t$. If follows that

$\mu_{\aleph}(0 * \tau) \geq \min\{\mu_{\aleph}(\rho * \tau), \mu_{\aleph}(0 * \rho)\} \geq t$

So that $0 * \tau \in U(\mu_{\aleph}, t)$, That mean $U(\mu_{\aleph}, t)$ is an RG-ideal of X.

In similarly, way can prove that $L(\nu_{\aleph}, s)$ is an RG-ideal of X.

$\rho * \tau \in L(\nu_{\aleph}, s)$ and $0 * \rho \in L(\nu_{\aleph}, s)$ and $\nu_{\aleph}(\rho * \tau) \leq s$ and $\nu_{\aleph}(0 * \rho) \leq s$

If follows that $\nu_{\aleph}(0 * \tau) \leq \max\{\nu_{\aleph}(\rho * \tau), \nu_{\aleph}(0 * \rho)\} \leq s$

So that $0 * \tau \in L(\nu_{\aleph}, s)$, that mean $L(\nu_{\aleph}, s)$ is an RG-ideal of X.■

Proposition 4.4. If An intuitionistic fuzzy subset $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ the sets $U(\mu_{\aleph}, t)$ and $L(\nu_{\aleph}, s)$ are RG-ideals of RG-algebra (X; *,0), for all t ∈ [0,1], then ℵ is an intuitionistic fuzzy subset is intuitionistic fuzzy RG-ideal of RG-algebra X.

Proof . Assume that for each t ∈[0,1] the set $U(\mu_\aleph, t)$ and $L(\nu_\aleph, s)$ are RG- ideals of X. For any $\rho \in X$ let that $\mu_\aleph(\rho) \geq t$ and $\nu_\aleph(\rho) \leq s$, then $\rho \in U(\mu_\aleph, t) \cap L(\nu_\aleph, s)$ and so $U(\mu_\aleph, t) \neq \emptyset \neq L(\nu_\aleph, s)$

Since $U(\mu_\aleph, t)$ and $L(\nu_\aleph, s)$ are RG-ideals of X

There for $0 \in U(\mu_\aleph, t) \cap L(\nu_\aleph, s)$ hence $\mu_\aleph(0) \geq t = \mu_\aleph(\rho), \nu_\aleph(0) \leq s = \nu_\aleph(\rho)$.

For all $\rho, \tau \in X$ if there exists such that $\mu_\aleph(0 * \tau) < \min\{\mu_\aleph(\rho * \tau), \mu_\aleph(0 * \rho)\}$

We taking $\acute{t} = \frac{1}{2}(\mu_\aleph(0 * \acute{\tau}) + \min\{\mu_\aleph(\rho' * \ \acute{\tau}), \mu_\aleph(0 * \acute{\rho}\ )\}$, we get

$\mu_\aleph(0 * \ \acute{\tau}) < t' < \min\{\mu_\aleph(\rho' * \ \acute{\tau}), \mu_\aleph(0 * \acute{\rho}\ )\}$ that mean $0 * \ \acute{\tau} \notin U(\mu_\aleph, \acute{t})$

$U(\mu_\aleph, \acute{t})$ is not RG-ideal of X leading to contradiction.

In other way, $\nu_\aleph(0 * \tau) > \max\{\nu_\aleph(\rho * \tau), \nu_\aleph(0 * \rho)\}$

We taking $\acute{s} = \frac{1}{2}(\nu_\aleph(0 * \acute{\tau}) + \max\{\nu_\aleph(\rho' * \ \acute{\tau}), \nu_\aleph(0 * \acute{\rho}\ )\})$, then

$\nu_\aleph(0 * \ \acute{\tau}) > \acute{s} > max\{\nu_\aleph(\rho' * \ \acute{\tau}), \nu_\aleph(0 * \acute{\rho}\ )\}$, since $0 * \ \acute{\tau} \ \notin L(\nu_\aleph, \acute{s}\ )$

$L(\nu_\aleph, \acute{s}\ )$ is not RG-ideal of X leading to contradiction.

Hence $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph\ (\rho)) \mid \rho \in X\}$ is an intuitionistic fuzzy RG-ideal of X.■

Proposition 4.5. Let $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph\ (\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG-ideal of RG-algebra (X; *,0), then ℵ is an intuitionistic fuzzy RG-subalgebra of X.

Proof . By Proposition (4.4) and Proposition (2.14) and Proposition (3.5). ■

Theorem 4.6. Let $\{\aleph_i | i = 1,2,3, \dots .\}$ be a family of intuitionistic fuzzy RG-ideal of RG-algebra (X; *,0), then the R-intersection of intuitionistic Ai is an intuitionistic fuzzy RG- RG-ideal of X is an intuitionistic fuzzy RG-ideal of X, where $\cap\, \aleph_i = (\inf\ \mu_{\aleph_i}(\rho)\, , \sup\, \nu_{\aleph_i}(\rho))$.

Proof . Let $\rho, \tau \in \cap_{\aleph_i}$, then $\rho, \tau\ \in \aleph_i$, for all $i \in \Lambda$

$\cap\, \mu_{\aleph_i}\, (0) = \cap\, \mu_{\aleph_i}\, (\rho * \rho) \geq \min\{\, \cap\, \mu_{\aleph_i}\, (\rho), \cap\, \mu_{\aleph_i}\, (\rho)\} = \cap\, \mu_{\aleph_i}\, (\rho)$,

$\cup\, \nu_{\aleph_i}(0) = \cup\, \nu_{\aleph_i}(\rho * \rho) \leq \max\{\, \cup\, \nu_{\aleph_i}(\rho), \cup\, \nu_{\aleph_i}(\rho)\} = \cup\, \nu_{\aleph_i}(\rho)$.

$\cap\, \mu_{\aleph_i}\, (0 * \tau) = \min\{\mu_{\aleph_i}\, (0 * y)\} \geq \min\{\min\{\mu_{\aleph_i}(\rho * \tau), \mu_{\aleph_i}\, (0 * \rho)\}\}$

$\geq \min\{\cap\, \mu_{\aleph_i}(\rho * \tau), \cap\, \mu_{\aleph_i}\, (0 * \rho)\}$ and

$\cup\, \nu_{\aleph_i}(0 * \tau) = \max\{\nu_{\aleph_i}\, (0 * y)\} \leq \max\{\max\{\nu_{\aleph_i}(\rho * \tau), \nu_{\aleph_i}\, (0 * \rho)\}\}$

$\leq \max\{\cup\, \nu_{\aleph_i}(\rho * \tau), \cup\, \nu_{\aleph_i}\, (0 * \rho)\}$.

Then R-intersection of intuitionistic $\aleph_i$is an intuitionistic fuzzy RG-ideal of X.■

Proposition 4.7. The P-intersection of any set $\aleph_i$ of intuitionistic subset of X, then P-intersection of $\aleph_i$ is an intuitionistic fuzzy RG-ideal of X, where $v_{\aleph_i}$ chain (Arterian).

Proof . By using Proposition (2.15(4)) and Proposition (2.24(4)). ■

Remark 4.8. If $v_{\aleph_i}$ is not chain in Proposition (4.7), then it is not true as the following example.

Example 4.9. Consider X in Example (3.2)

X	0	1	2	3
$\mu_\aleph$	0.9	0.9	0.2	0.2
μ_B	0.7	0.1	0.7	0.1
$\mu_\aleph \cup \mu_B$	0.9	0.9	0.7	0.2
$v_\aleph$	0.2	0.2	0.4	0.4
v_B	0.3	0.5	0.3	0.5
$v_\aleph \cap v_B$	0.2	0.2	0.3	0.4

It is easy to show that. inf $\mu_{\aleph_i}(\rho)$ is a fuzzy RG-ideal, but $v_\aleph \cap v_B$ is not doubt fuzzy RG-ideal of X, since $\rho = 2, \tau = 3, v_\aleph \cap v_B(0 * 3) = 0.4 \not\leq \max\{v_\aleph \cap v_B(2 * 3), v_\aleph \cap v_B(0 * 2)\} = 0.3$ and $v_B(3) = 0.4 \not\leq \max\{v_\aleph \cap v_B(1), v_\aleph \cap v_B(2)\} = 0.3$

Proposition 4.10. The P- union of $\aleph_i$ of intuitionistic subset of X, then the P-union of $\aleph_i$ is an intuitionistic fuzzy RG-ideal of X, where $\mu_{\aleph_i}$ be a chain (Noetherian)

Proof . If $\mu_{\aleph_i}$ is a chain (Noetherian), by using Proposition (2.24(3)) The union of any set of fuzzy RG-ideals of RG-algebra $(X; *, 0)$ is also fuzzy RG-ideal of X, if μ_i is chain.

By Proposition (2.24(4)) the union of any set of doubt fuzzy RG -ideals of RG-algebra is also doubt fuzzy RG-ideal.

Then P-union of $\aleph_i$ is an intuitionistic fuzzy RG-ideal of X. ■

Remark 4.11. If $\mu_{\aleph_i}$ is not chain in Proposition (4.10), then it is not true as the following example.

Example 4.12. Consider the following Example. (4.9), the $(\mu_\aleph \cup \mu_B)$ are not fuzzy RG-ideal of X,

let ρ=1, τ=3 then we have

$(\mu_\aleph \cup \mu_B)(0 * 3) = (\mu_\aleph \cup \mu_B)(3) \geq \min\{(\mu_\aleph \cup \mu_B)(1 * 3), (\mu_\aleph \cup \mu_B)(0 * 1)\}$

$(\mu_\aleph \cup \mu_B)(3) = 0.2 \ngeq 0.7 = \min\{(\mu_\aleph \cup \mu_B)(2), (\mu_\aleph \cup \mu_B)(1)\}$

Proposition 4.13. The R-union of $\aleph_i$ of intuitionistic subset of a RG-algebra (X; *,0), then R-union of $\aleph_i$ is an intuitionistic fuzzy RG-ideal of X, where $\mu_{\aleph_i}$ (Noetherian) is chain and $v_{\aleph_i}$ chain(Arterian)

Proof . By using Proposition (2.15) the union of any set of fuzzy RG-ideals of RG-algebra $(X; *, 0)$ is also fuzzy RG-ideal of X, if μ_i is chain (Noetherian). We have $\cup\, \mu_{\aleph_i}$ is fuzzy RG-ideal of X and by using Proposition (2.24), the intersection of any set of doubt fuzzy RG-ideals of RG-algebra $(X; *, 0)$ is also doubt fuzzy RG-ideal of X where is chain (Arterian). $\cap\, v_{\aleph_i}$ is doubt fuzzy RG-ideal.

Hence the R-union of $\aleph_i$ is intuitionistic fuzzy RG-ideal of X. ■

Remark 4.14. If $v_{\aleph_i}$ or $\mu_{\aleph_i}$ are not (chain (Arterian), chain (Notherian)) respectively in Proposition (4.13), then it is not true as the following example.

Example 4.15. Consider X in Example (3.2)

X	0	1	2	3
$\mu_\aleph$	0.9	0.9	0.2	0.2
μ_B	0.7	0.1	0.7	0.1
$\mu_\aleph \cup \mu_B$	0.9	0.9	0.7	0.2
$v_\aleph$	0.2	0.2	0.4	0.4
v_B	0.3	0.5	0.3	0.5
$v_\aleph \cap v_B$	0.2	0.2	0.3	0.4

Then $(\mu_\aleph \cup \mu_B)$ are not fuzzy RG-ideal of X, since

$\mu_\aleph \cup \mu_B(0 * 3) = \mu_\aleph \cup \mu_B(3) = 0.2 \ngeq 0.7=\min\{(\mu_\aleph \cup \mu_B)(1 * 3), (\mu_\aleph \cup \mu_B)(0 * 1)\}$

$\mu_\aleph \cup \mu_B(3) = 0.2 \ngeq 0.7=\min\{(\mu_\aleph \cup \mu_B)(2), (\mu_\aleph \cup \mu_B)(1)\}$

And $v_\aleph \cap v_B$ is not doubt fuzzy RG-ideal of X since $\rho = 2, \tau = 3$,

$(v_\aleph \cap v_B)(0 * 3) = 0.4 \leq \max\{(v_\aleph \cap v_B)(2 * 3), (v_\aleph \cap v_B)(0 * 2)\} = 0.3$

$(v_A \cap v_B)(3) = 0.4 \nleq \max\{(v_A \cap v_B)(1), (v_A \cap v_B)(2)\} = 0.3$

Theorem 4.16. Let $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG-ideal of X if and only if the fuzzy set $\mu_\aleph(\rho)$ and $\bar{v}_\aleph(\rho)$ are fuzzy RG-ideal of X.

Proof . Since $\aleph = \{(\rho, \mu_\aleph(\rho), \nu_\aleph(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG-ideal. Cleary, $\mu_\aleph(\rho)$ is a fuzzy RG-ideal of X. For all $\rho, \tau \in X$, we have

$\bar{v}_\aleph(0) = 1 - v_\aleph(0) \geq 1 - v_\aleph(\rho) = \bar{v}_\aleph(\rho)$

$\bar{v}_\aleph(0 * \tau) = 1 - v_\aleph(0 * \tau) \geq 1 - \max\{\nu_\aleph(\rho * \tau), \nu_\aleph(0 * \rho)\}$

$\geq \min\{1 - \nu_\aleph(\rho * \tau), 1 - \nu_\aleph(0 * \rho)\} \geq \min\{\bar{v}_\aleph(\rho * \tau), \bar{v}_\aleph(0 * \rho)\}$.

Hence $\bar{v}_\aleph$ is fuzzy RG-ideal of X.

The conversely, assume that $\mu_\aleph, \bar{v}_\aleph$ are fuzzy RG-ideal of X, for every $\rho, \tau \in X$,we get

$\mu_\aleph(0) \geq \mu_\aleph(\rho), 1 - \nu_\aleph(0) = \bar{v}_\aleph(0) \geq \bar{v}_\aleph(\rho) = 1 - \nu_\aleph(\rho)$ that is $\nu_\aleph(0) \leq \nu_\aleph(\rho)$, $\mu_\aleph(0 * \tau) \geq \min\{\mu_\aleph(\rho * \tau), \mu_\aleph(0 * \rho)\}$ and $1 - \nu_\aleph(0 * \tau) = \bar{v}_\aleph(0 * \tau)$

$\geq \min\{\bar{v}_\aleph(\rho * \tau), \bar{v}_\aleph(0 * \rho)\} = \min\{1 - \nu_\aleph(\rho * \tau), 1 - \nu_\aleph(0 * \rho)\}$

$=1-\max\{\nu_{\aleph}(\rho*\tau), \nu_{\aleph}(0*\rho)\}$

That is $\nu_{\aleph}(0*\tau) \leq \max\{\nu_{\aleph}(\rho*\tau), \nu_{\aleph}(0*\rho)\}$.

Hence $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ an intuitionistic fuzzy RG-ideal. ■

Theorem 4.17. Let f: $(X;*,0) \to (Y;*',0')$be a homomorphism of RG-algebra if $B = (\mu_B(\rho), \nu_B(\rho))$ is an intuitionistic fuzzy RG-ideal of Y with sup and inf properties, then the pre-image $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B)$ of B under f in X is an intuitionistic fuzzy RG-ideal of X.

Proof . For all $\rho \in X$, $f^{-1}(\mu_B)(\rho) = \mu_B(f(\rho)) \leq \mu_B(f(0)) = \mu_B(\acute{0}) = f^{-1}(\mu_B)(0)$

$f^{-1}(\nu_B)(\rho) = \nu_B(f(\rho)) \geq \nu_B(f(0)) = \nu_B(\acute{0}) = f^{-1}(\nu_B)(0)$.

Let $\rho, \tau \in X$, then $f^{-1}(\mu_B)(0*\tau) = \mu_B(f(0*\tau)) = \mu_B(f(0) \acute{*} f(\tau))$

$\geq \min\{\mu_B(f(\rho) \acute{*} f(\tau)), \mu_B(f(0) \acute{*} f(\rho))\} = \min\{\mu_B(f(\rho*\tau)), \mu_B(f(0*\rho))\}$

$=\min\{f^{-1}(\mu_B)(\rho*\tau), f^{-1}(\mu_B)(0*\rho)\}$

and $f^{-1}(\nu_B)(0*\tau) = \nu_B(f(0*\tau)) = \nu_B(f(0) \acute{*} f(\tau))$

$\leq \max\{\nu_B(f(\rho) \acute{*} f(\tau)), \nu_B(f(0) \acute{*} f(\rho))\}$

$=\max\{\nu_B(f(\rho*\tau)), \nu_B(f(0*\rho))\}$

$=\max\{f^{-1}(\nu_B)(\rho*\tau), f^{-1}(\nu_B)(0*\rho)\}$.

Hence $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B)$ of B under f in X is an intuitionistic fuzzy RG-ideal of X.

Theorem 4.18. Let f: $(X;*,0) \to (Y;\acute{*},\acute{0})$be epimorphism of RG-algebras, if $\aleph = (\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho))$ is an intuitionistic fuzzy RG-ideal of X with sup and inf properties, then $f(\aleph) = (\tau, f(\mu_{\aleph})(y), f(\nu_{\aleph})(\tau))$ of $\aleph$ is an intuitionistic fuzzy RG-ideal of Y.

Proof.

1) For all $\rho \in X$, there exists $\tau \in Y$ such that f (ρ)=τ. Since $\mu_{\aleph}(0) \geq \mu_{\aleph}(\rho)$, then $f(\mu_{\aleph})(0) \geq f(\mu_{\aleph})(\tau)$ and $\nu_{\aleph}(0) \leq \nu_{\aleph}(\rho)$, then $f(\nu_{\aleph})(0) \leq f(\nu_{\aleph}(a))$.

2) Let $a, b \in X$, then $\rho, \tau \in Y$ such that f (a)=ρ, f (b)=τ, and $f(a*b) = x*'y$

And $f(0*a) = 0'*'x$, and $f(0*b) = 0'*'y$.

$\mu_{\aleph}^{f}(\rho) = \mu_{\aleph}(f(a))$, $\mu_{\aleph}^{f}(\tau) = \mu_{\aleph}(f(b))$, and $\mu_{\aleph}^{f}(\rho*\tau) = \mu_{\aleph}(f(a*b))$,

$\mu_{\aleph}^{f}(0'*'\tau) = \mu_{\aleph}(f(0)*'f(b)) = \mu_{\aleph}(f(0*b))$

$\geq \min\{\mu_{\aleph}(f(a*b)), \mu_{\aleph}(f(0*\rho))\}$

$=\min\{\mu_{\aleph}(f(a)*'f(b)), \mu_{\aleph}(f(0)*'f(\rho))\}$

$=\min\{\mu^f_{\aleph}(\rho *'\tau), \mu^f_{\aleph}(0' *' x)\}$ and

$\nu^f_{\aleph}(\rho) = \nu_{\aleph}(f(a)), \nu^f_{\aleph}(\tau) = \nu_{\aleph}(f(b))$, and $\nu^f_{\aleph}(\rho * \tau) = \nu_{\aleph}(f(a * b))$,

$$\nu^f_{\aleph}(0' *' \tau) = \nu_{\aleph}(f(0) *' f(b)) = \nu_{\aleph}(f(0 * b)) \leq \max\{\nu_{\aleph}(f(a * b)), \nu_{\aleph}(f(0 * \rho))\}$$

$=\max\{\nu_{\aleph}(f(a) *' f(b)), \nu_{\aleph}(f(0) *' f(\rho))\} = \min\{\nu^f_{\aleph}(\rho *'\tau), \nu^f_{\aleph}(0' *' x)\}$ and

Hence $f(\aleph) = (f(\mu_{\aleph}), f(\nu_{\aleph})$ of $\aleph$ is an intuitionistic fuzzy RG-ideal of Y. ■

Proposition 4.19. Every intuitionistic fuzzy RG-ideal of RG-algebra $(X; *, 0)$ is an intuitionistic fuzzy RG-subalgebra of X.

Proof . By using Proposition (2.24(4)) and Proposition (2.14).

Remark 4.20. The converse of Proposition (4.19) is not true as the following example:

Example 4.21. Consider X in Example (3.2)

Then $\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ be an intuitionistic fuzzy RG-subalgebra of X

when $\mu(x)$ is fuzzy RG-ideal $\mu(x) = \begin{cases} 0.8 & x = 0 \\ 0.3 & x \in \{1,2,3\} \end{cases}$ and $\nu(\rho)$ is a doubt fuzzy RG-subalgebra of X $\nu(\rho) = \begin{cases} 0.3 & \rho \in \{0,3\} \\ 0.8 & \rho = 1 \\ 0.9 & \rho = 2 \end{cases}$.

But $\nu(\rho)$ is not a doubt fuzzy RG-ideal since Let ρ=1, τ=2 then $\nu(0 * 2) = 0.9 \not\leq \max\{\nu(1 * 2), \nu(0 * 1)\} = 0.8$.

$\aleph = \{(\rho, \mu_{\aleph}(\rho), \nu_{\aleph}(\rho)) \mid \rho \in X\}$ is not intuitionistic fuzzy RG-ideal of RG-algebra

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