The Two Techniques (SVD) And (LU) and Linking them Through Mathematical Logic

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Abstract
We have previously conducted a scientific research on the relationship between the matrices of two matrix analyzes, namely (SVD) and (GSVD) for the two matrices($A_{(m,p)}$ and $B_{(n,p)}$). Where we showed that the matrices of these two techniques have a specific algebraic relationship that benefits researchers in the field of image optimization, encoding and image processors as the researcher wants, the research has been published in an international journal[1].The above research included in its conclusion an indication of future work, the first of which is the content of this title of our research in which we will find the relationship between technical matrices (SVD). And the (LU) technique, which are two matrix analyzes ($C_{(t,h)}$ and $D_{(r,h)}$). The results of this research paper will have a relationship with the results of the research referred to above, as we will derive through these two results the relationship between (GSVD) technology and (LU) technology because there will be a common factor between them, which is the (SVD) technology. Thus, we will have three important relationships: the relationship between (SVD) and (GSVD), the relationship between (SVD) and (LU), and the relationship between (GSVD) and (LU). These relationships will provide excellent mathematical tools in the field of image processing and other related work.

Keywords: SVD, LU.

1. Introduction

Matrices and their science has greatly evolved and become a major part of all sciences. Where everything can be simulated in the form of a real or complex numerical matrix or any possible object and work on this matrix as required by the problem under discussion and research. And the digital world is the best proof of the validity of this saying, as we find the matrix is the being that is the soul of everything in the universe. It has entered the matrix widely in pure sciences, astronomy and images through processors, compression, and coding in mathematical algebraic and non-algebraic methods. We do not forget the mathematicians who developed the science of matrices over the years, as they invented the techniques of matrix analyzes, which are considered one of the greatest sciences that pushed the scientific renaissance to great levels, especially after the discovery of the computer and its development, as well as the Internet, which represented the biggest shift in all sciences without exception.

It is these technologies, but not all of them, (SVD) and (GSVD) technology, which we have published a scientific paper explaining the relationship between their arrays. As well as the (LU) technology, which we will conclude through our research of this relationship between its matrices and (SVD) arrays, and because the (SVD) technology is a common factor between the two technologies (GSVD) and (LU). Therefore, from the results of our research this is the relationship between the matrices of each of (GSVD) and (LU).

Very clear and by defining the relationship between the arrays of all incoming technologies, including the (QR) technique, the scheme is to write a paper on the relationship between its matrices and the matrices of the rest of the techniques. This work will certainly give great flexibility in dealing with image arrays within the field of image processors of all kinds.

2. Research Idea:

We indicated in our previous research that the main idea from researches program is to find the relationship between matrices of any two matrix analyzes. We have come a simple way in this goal, which is to find the relationship between matrices ((SVD) and (GSVD)] through previous research published in one of the international journals[1] and it is considered a good start in this aspect, and now in our research we will examine the relationship between matrices [(SVD) & (LU)). It is the idea of our research, and work is underway in the future to find the relationship between the matrices of any of the remaining two matrix analyzes ([(SVD) & (QR)], [(GSVD) & (LU)], [(GSVD) & (QR)], and [(LU) & (QR)]). In addition to the rest of the matrix analyzes such as (chol), (ilu), (gf)... etc.

3. A BRIEF EXPLANATION OF THE TWO ANALYZES (SVD) & (LU).

3.1. Singular Values Decomposition (SVD)[2][3][4][5][6][7][8]:-

Definition:-

$\forall\ \mathrm{K}_{(c,z)}$ is real matrix

$\exists\ \mathrm{L}_{(c,c)} \& \mathrm{H}_{(z,z)}$ are orthogonally metrices

$\ni\ \left[\mathrm{L}_{(c,c)}, \mathrm{S}_{(c,z)}, \mathrm{H}_{(z,z)}\right] = \mathrm{svd}\left(\mathrm{K}_{(c,z)}\right)$

$\Longrightarrow \mathrm{K}_{(c,z)} = \mathrm{L}_{(c,c)} * \mathrm{S}_{(c,z)} * {\mathrm{H}_{(z,z)}}^{\mathrm{T}}$

$\mathrm{S}_{(c,z)}$ is non diagonal entries all zero

$\&\ s_{11} \geq s_{22} \geq \cdots \geq s_{jj} \geq 0$ where $j = \min\{c, z\}$

And :-

$s_{(i,i)} \forall\ 1 \leq i \leq \min\{c, z\}$ are called the singular values of K.

$\mathrm{L}_{(:,i)}\ 1 \leq i \leq c$ are called the left singular vectors of K.

$\mathrm{H}_{(:,j)}\ 1 \leq j \leq z$ are called the right singular vectors of K.

$s_{(i,i)} \forall\ 1 \leq i \leq \min\{c, z\}$

$= \sqrt{\text{are the square roots of the eigenvalues of the symmetrical square matrix } \left(\left(\mathrm{K}_{(c,z)}\right)^{\mathrm{T}} * \mathrm{K}_{(c,z)}\right)}.$

$\mathrm{H}_{(:,j)}\ 1 \leq j \leq z$ are the eigenvectors of $\left(\left(\mathrm{K}_{(c,z)}\right)^{\mathrm{T}} * \mathrm{K}_{(c,z)}\right)$.

$$\mathrm{L}_{(:,i)}\ 1 \leq i \leq c = \frac{1}{s_{ii}} * \mathrm{K}_{(c,z)} * \mathrm{H}_{(:,i)}\ 1 \leq i \leq z$$

And by Gram-Schmidt method we can convert two matrices arrays ($\mathrm{L}_{(c,c)}$ & $\mathrm{H}_{(z,z)}$) to an orthogonal matrices and we can divide each column along its length to turn the two matrices into orthonormal.
To find out more details can go back to the references contained in the conclusion of this research.

3.2. An Illustrative Example of (svd)

$$Let\ K_{(7,5)} = \begin{bmatrix} 21\ 45\ 78\ 36\ 69 \\ 85\ 74\ 96\ 82\ 10 \\ 91\ 73\ 64\ 31\ 97 \\ 22\ 11\ 33\ 44\ 55 \\ 95\ 51\ 75\ 53\ 91 \\ 51\ 57\ 59\ 53\ 58 \\ 20\ 30\ 40\ 50\ 90 \end{bmatrix}$$

Then [$L_{(7,7)}$, $S_{(7,5)}$, $H_{(5,5)}$]=svd($K_{(7,5)}$)

$L_{(7,7)}$ =

-0.3267 -0.1716 0.4416 -0.7148 0.3952 0.0022 -0.0415
-0.4336 0.8090 0.2187 0.0973 -0.0565 -0.2633 0.1665
-0.4653 -0.1758 -0.6278 -0.2779 -0.2747 -0.4302 -0.1440
-0.2143 -0.2232 0.3071 0.4855 0.1952 -0.3477 -0.6444
-0.4764 -0.0822 -0.3441 0.3301 0.5744 0.3928 0.2337
-0.3551 0.0470 0.1126 -0.0338 -0.5044 0.6739 -0.3864
-0.2997 -0.4758 0.3710 0.2378 -0.3776 -0.1278 0.5750

$H_{(5,5)}$ =

-0.4592 0.3141 -0.7182 0.3686 0.1970
-0.3918 0.2256 -0.1093 -0.5047 -0.7273
-0.4950 0.2489 0.3599 -0.4491 0.6015
-0.3734 0.2060 0.5836 0.6381 -0.2654
-0.5011 -0.8637 -0.0468 0.0250 -0.0083

$S_{(7,5)}$ =

348.7043 0 0 0 0
0 93.3759 0 0 0
0 0 58.5370 0 0
0 0 0 35.4016 0
0 0 0 0 20.7460
0 0 0 0 0
0 0 0 0 0

s.t.

$$\left(L_{(7,7)}\right) * \left(S_{(7,5)}\right) * \left(H_{(5,5)}\right)^T = K_{(7,5)}$$

3.3. Lower and upper triangular matrix (LU)[9][10][11][12][13][14][15][16][17].

LU function:-
Let [X,Y]=lu(Z)
X is (Bottom trigonometric matrix)
Y is (Supreme trigonometric matrix)
Such that Z=X*Y
[X,Y,T] = lu(Z)
X is (Bottom trigonometric matrix)
Y is (Supreme trigonometric matrix)
T is (Matrix reciprocal)
Such that T*Z=X*Y
[X,Y,q] = lu(Z,'v') v is vector.
X is (Bottom trigonometric matrix)
Y is (Supreme trigonometric matrix)
q is (row vector)
Such that Z(q,:)=X*Y
[X,Y,Q] = lu(Z,'M') M is matrix.

X is (Bottom trigonometric matrix)
Y is (Supreme trigonometric matrix)
Q is (permutation matrix)
Such that Z(q,:)=X*Y

And because of the problems of the conditions of electronic plagiarism that did not preserve value for scientific research, as arbitrary controls that terminate scientific research by simply increasing the percentage of electronic plagiarism, regardless of the originality of the idea and the quality of the work or the algorithm. We are satisfied with what has been explained about the (LU) analysis technique, according to the need of our research, and we leave the reader who wishes to increase his information about this technology by returning to the references at the end of the research, which include adequate explanations about the details and types of this technology.

We will discuss the relationship within one type of (lu) analysis.

3.2.1. An illustrative example of (lu):-

$$Let\ KK_{(7,5)} = \begin{bmatrix} 21\ 45\ 78\ 36\ 69 \\ 85\ 74\ 96\ 82\ 10 \\ 91\ 73\ 64\ 31\ 97 \\ 22\ 11\ 33\ 44\ 55 \\ 95\ 51\ 75\ 53\ 91 \\ 51\ 57\ 59\ 53\ 58 \\ 20\ 30\ 40\ 50\ 90 \end{bmatrix}$$

Then $[LL_{(7,5)}, SS_{(5,5)}, HH_{(7,7)}]$=lu(K$K_{(7,5)}$)

$LL_{(7,5)} =$

1.0000 0 0 0 0
0.2211 1.0000 0 0 0
0.9579 0.7160 1.0000 0 0
0.2105 0.5712 0.2098 1.0000 0
0.8947 0.8411 0.4394 0.9302 1.0000
0.5368 0.8783 0.6794 0.8687 0.4002
0.2316 -0.0240 -0.3301 0.6117 0.0184

$HH_{(7,7)} =$

0 0 0 0 1 0 0
1 0 0 0 0 0 0
0 0 1 0 0 0 0
0 0 0 0 0 0 1
0 1 0 0 0 0 0
0 0 0 0 0 1 0
0 0 0 1 0 0 0

$SS_{(5,5)} =$

95.0000 51.0000 75.0000 53.0000 91.0000
0 33.7263 61.4211 24.2842 48.8842
0 0 -51.8184 -37.1554 -25.1685
0 0 0 32.7666 48.2014
0 0 0 0 -146.3159

such that $\left(HH_{(7,7)}\right)^T * \left(LL_{(7,5)}\right) * \left(SS_{(5,5)}\right) = KK_{(7,5)}$

4. THE TWO TECHNIQUES (SVD) AND (LU) AND LINKING THEM THROUGH MATHEMATICAL LOGIC.

Let $K_{(n,m)}$ be a real matrix.

$[L_{(n,n)}, S_{(n,m)}, H_{(m,m)}]$=svd($K_{(n,m)}$)

$[LL_{(n,m)}, SS_{(m,m)}, HH_{(n,n)}]$=lu($K_{(n,m)}$)

From analysis SVD :-

$$\left(L_{(n,n)}\right) * \left(S_{(n,m)}\right) * \left(H_{(m,m)}\right)^T = K_{(n,m)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From analysis LU:-

$$\left(HH_{(n,n)}\right)^T * \left(LL_{(n,m)}\right) * \left(SS_{(m,m)}\right) = K_{(n,m)} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From the functions (1) and (3):-

$$\left(L_{(n,n)}\right) * \left(S_{(n,m)}\right) * \left(H_{(m,m)}\right)^T = \left(HH_{(n,n)}\right)^T * \left(LL_{(n,m)}\right) * \left(SS_{(m,m)}\right) \ldots\ldots (3)$$

$$S_{(n,m)} = \left(L_{(n,n)}\right)^T * \left(HH_{(n,n)}\right)^T * \left(LL_{(n,m)}\right) * \left(SS_{(m,m)}\right) * \left(H_{(m,m)}\right)$$

$$S_{(n,m)} = \begin{bmatrix} L_{(1,1)} & L_{(1,2)} & \cdots & L_{(1,n)} \\ L_{(2,1)} & L_{(2,2)} & \cdots & L_{(2,n)} \\ \vdots & \vdots & \vdots & \vdots \\ L_{(n,1)} & L_{(n,2)} & \cdots & L_{(n,n)} \end{bmatrix}^T * \begin{bmatrix} HH_{(1,1)} & HH_{(1,2)} & \cdots & HH_{(1,n)} \\ HH_{(2,1)} & HH_{(2,2)} & \cdots & HH_{(2,n)} \\ \vdots & \vdots & \vdots & \vdots \\ HH_{(n,1)} & HH_{(n,2)} & \cdots & HH_{(n,n)} \end{bmatrix}^T * \left(LL_{(n,m)}\right)$$
$$* \begin{bmatrix} SS_{(1,1)} & SS_{(1,2)} & \cdots & SS_{(1,m)} \\ SS_{(2,1)} & SS_{(2,2)} & \cdots & SS_{(2,m)} \\ \vdots & \vdots & \vdots & \vdots \\ SS_{(m,1)} & SS_{(m,2)} & \cdots & SS_{(m,m)} \end{bmatrix} * \begin{bmatrix} H_{(1,1)} & H_{(1,2)} & \cdots & H_{(1,m)} \\ H_{(2,1)} & H_{(2,2)} & \cdots & H_{(2,m)} \\ \vdots & \vdots & \vdots & \vdots \\ H_{(m,1)} & H_{(m,2)} & \cdots & H_{(m,m)} \end{bmatrix}$$

$$S_{(n,m)} = \begin{bmatrix} L_{(1,1)} & L_{(2,1)} & \cdots & L_{(n,1)} \\ L_{(1,2)} & L_{(2,2)} & \cdots & L_{(n,2)} \\ \vdots & \vdots & \vdots & \vdots \\ L_{(1,n)} & L_{(2,n)} & \cdots & L_{(n,n)} \end{bmatrix} * \begin{bmatrix} HH_{(1,1)} & HH_{(2,1)} & \cdots & HH_{(n,1)} \\ HH_{(1,2)} & HH_{(2,2)} & \cdots & HH_{(n,2)} \\ \vdots & \vdots & \vdots & \vdots \\ HH_{(1,n)} & HH_{(2,n)} & \cdots & HH_{(n,n)} \end{bmatrix} * \left(LL_{(n,m)}\right)$$
$$* \begin{bmatrix} SS_{(1,1)} & SS_{(1,2)} & \cdots & SS_{(1,m)} \\ SS_{(2,1)} & SS_{(2,2)} & \cdots & SS_{(2,m)} \\ \vdots & \vdots & \vdots & \vdots \\ SS_{(m,1)} & SS_{(m,2)} & \cdots & SS_{(m,m)} \end{bmatrix} * \begin{bmatrix} H_{(1,1)} & H_{(1,2)} & \cdots & H_{(1,m)} \\ H_{(2,1)} & H_{(2,2)} & \cdots & H_{(2,m)} \\ \vdots & \vdots & \vdots & \vdots \\ H_{(m,1)} & H_{(m,2)} & \cdots & H_{(m,m)} \end{bmatrix}$$

$$S_{(n,m)} = \begin{bmatrix} \sum_{i=1}^{n} L_{(i,1)} * HH_{(1,i)} & \sum_{i=1}^{n} L_{(i,1)} * HH_{(2,i)} & \cdots & \sum_{i=1}^{n} L_{(i,1)} * HH_{(n,i)} \\ \sum_{i=1}^{n} L_{(i,2)} * HH_{(1,i)} & \sum_{i=1}^{n} L_{(i,2)} * HH_{(2,i)} & \cdots & \sum_{i=1}^{n} L_{(i,2)} * HH_{(n,i)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{n} L_{(i,n)} * HH_{(1,i)} & \sum_{i=1}^{n} L_{(i,n)} * HH_{(2,i)} & \cdots & \sum_{i=1}^{n} L_{(i,n)} * HH_{(n,i)} \end{bmatrix} * \left(LL_{(n,m)}\right)$$
$$* \begin{bmatrix} \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,m)} \\ \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,m)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,m)} \end{bmatrix}$$

$$S_{(n,m)} = \begin{bmatrix} \sum_{i=1}^{n} L_{(i,1)} * HH_{(1,i)} & \sum_{i=1}^{n} L_{(i,1)} * HH_{(2,i)} & \cdots & \sum_{i=1}^{n} L_{(i,1)} * HH_{(n,i)} \\ \sum_{i=1}^{n} L_{(i,2)} * HH_{(1,i)} & \sum_{i=1}^{n} L_{(i,2)} * HH_{(2,i)} & \cdots & \sum_{i=1}^{n} L_{(i,2)} * HH_{(n,i)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^{n} L_{(i,n)} * HH_{(1,i)} & \sum_{i=1}^{n} L_{(i,n)} * HH_{(2,i)} & \cdots & \sum_{i=1}^{n} L_{(i,n)} * HH_{(n,i)} \end{bmatrix} * \begin{bmatrix} LL_{(1,1)} & LL_{(1,2)} & \cdots & LL_{(1,m)} \\ LL_{(2,1)} & LL_{(2,2)} & \cdots & LL_{(2,m)} \\ \vdots & \vdots & \vdots & \vdots \\ LL_{(n,1)} & LL_{(n,2)} & \cdots & LL_{(n,m)} \end{bmatrix}$$

$$* \begin{bmatrix} \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,m)} \\ \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,m)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,m)} \end{bmatrix}$$

$$S_{(n,m)}$$

$$= \begin{bmatrix} \sum_{p=1}^{n} \left\{ LL_{(p,1)} * \sum_{i=1}^{n} L_{(i,1)} * HH_{(p,i)} \right\} & \sum_{p=1}^{n} \left\{ LL_{(p,2)} * \sum_{i=1}^{n} L_{(i,1)} * HH_{(p,i)} \right\} & \cdots & \sum_{p=1}^{n} \left\{ LL_{(p,m)} * \sum_{i=1}^{n} L_{(i,1)} * HH_{(p,i)} \right\} \\ \sum_{p=1}^{n} \left\{ LL_{(p,1)} * \sum_{i=1}^{n} L_{(i,2)} * HH_{(p,i)} \right\} & \sum_{p=1}^{n} \left\{ LL_{(p,2)} * \sum_{i=1}^{n} L_{(i,2)} * HH_{(p,i)} \right\} & \cdots & \sum_{p=1}^{n} \left\{ LL_{(p,m)} * \sum_{i=1}^{n} L_{(i,2)} * HH_{(p,i)} \right\} \\ & \vdots & \vdots & \vdots \\ \sum_{p=1}^{n} \left\{ LL_{(p,1)} * \sum_{i=1}^{n} L_{(i,n)} * HH_{(p,i)} \right\} & \sum_{p=1}^{n} \left\{ LL_{(p,2)} * \sum_{i=1}^{n} L_{(i,n)} * HH_{(p,i)} \right\} & \cdots & \sum_{p=1}^{n} \left\{ LL_{(p,m)} * \sum_{i=1}^{n} L_{(i,n)} * HH_{(p,i)} \right\} \end{bmatrix}$$

$$* \begin{bmatrix} \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(1,j)} * H_{(j,m)} \\ \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(2,j)} * H_{(j,m)} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,1)} & \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,2)} & \cdots & \sum_{j=1}^{m} SS_{(m,j)} * H_{(j,m)} \end{bmatrix}$$

$$S_{(n,m)}$$

$$= \begin{bmatrix} \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,1)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,1)} \right) \right] & \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,1)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,2)} \right) \right] & \cdots & \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,1)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,m)} \right) \right] \\ \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,2)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,1)} \right) \right] & \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,2)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,2)} \right) \right] & \cdots & \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,2)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,m)} \right) \right] \\ & \vdots & \vdots & \vdots \\ \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,n)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,1)} \right) \right] & \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,n)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,2)} \right) \right] & \cdots & \sum_{t=1}^{m} \left[\left(\sum_{p=1}^{n} \left\{ LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,n)} * HH_{(p,i)} \right\} \right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,m)} \right) \right] \end{bmatrix}$$

$$\left.\begin{array}{l}\sum_{t=1}^{m}\left[\left(\sum_{p=1}^{n}\left\{LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,1)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,1)}\right)\right] = S_{(1,1)} \\ \therefore \sum_{t=1}^{m}\left[\left(\sum_{p=1}^{n}\left\{LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,2)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,2)}\right)\right] = S_{(2,2)} \\ \vdots \\ \sum_{t=1}^{m}\left[\left(\sum_{p=1}^{n}\left\{LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,q)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,q)}\right)\right] = S_{(q,q)}\end{array}\right\} \quad q = min\{n, m\}$$

$$\& \sum_{t=1}^{m}\left[\left(\sum_{p=1}^{n}\left\{LL_{(p,t)} * \sum_{i=1}^{n} L_{(i,w)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{m} SS_{(t,j)} * H_{(j,z)}\right)\right] = 0 \ \forall w \neq z$$

The last equations represent the mathematical relationship between the singular values extracted by matrix analysis (SVD) of the$K_{(n,m)}$ matrix and the numerical values of the matrix analysis (LU).

The numerical application of the results:-
Illustrative example
We return to our previous example above:

$$Let\ \boldsymbol{K}_{(\mathbf{7},\mathbf{5})} = \begin{bmatrix} 21\ 45\ 78\ 36\ 69 \\ 85\ 74\ 96\ 82\ 10 \\ 91\ 73\ 64\ 31\ 97 \\ 22\ 11\ 33\ 44\ 55 \\ 95\ 51\ 75\ 53\ 91 \\ 51\ 57\ 59\ 53\ 58 \\ 20\ 30\ 40\ 50\ 90 \end{bmatrix}$$

Then $[\boldsymbol{L}_{(\mathbf{7},\mathbf{7})}, \boldsymbol{S}_{(\mathbf{7},\mathbf{5})}, \boldsymbol{H}_{(\mathbf{5},\mathbf{5})}]$=svd($\boldsymbol{K}_{(\mathbf{7},\mathbf{5})}$)

$\boldsymbol{L}_{(\mathbf{7},\mathbf{7})} =$

-0.3267 -0.1716 0.4416 -0.7148 0.3952 0.0022 -0.0415
-0.4336 0.8090 0.2187 0.0973 -0.0565 -0.2633 0.1665
-0.4653 -0.1758 -0.6278 -0.2779 -0.2747 -0.4302 -0.1440
-0.2143 -0.2232 0.3071 0.4855 0.1952 -0.3477 -0.6444
-0.4764 -0.0822 -0.3441 0.3301 0.5744 0.3928 0.2337
-0.3551 0.0470 0.1126 -0.0338 -0.5044 0.6739 -0.3864
-0.2997 -0.4758 0.3710 0.2378 -0.3776 -0.1278 0.5750

$\boldsymbol{H}_{(\mathbf{5},\mathbf{5})} =$

-0.4592 0.3141 -0.7182 0.3686 0.1970
-0.3918 0.2256 -0.1093 -0.5047 -0.7273
-0.4950 0.2489 0.3599 -0.4491 0.6015
-0.3734 0.2060 0.5836 0.6381 -0.2654
-0.5011 -0.8637 -0.0468 0.0250 -0.0083

$\boldsymbol{S}_{(\mathbf{7},\mathbf{5})} =$

348.7043 0 0 0 0
0 93.3759 0 0 0
0 0 58.5370 0 0
0 0 0 35.4016 0
0 0 0 0 20.7460
0 0 0 0 0

0 0 0 0 0

s.t.

$\left(\boldsymbol{L}_{(7,7)}\right) * \left(\boldsymbol{S}_{(7,5)}\right) * \left(\boldsymbol{H}_{(5,5)}\right)^{\boldsymbol{T}} = \boldsymbol{K}_{(7,5)}$

And :-

Then [$\boldsymbol{LL}_{(7,5)}$, $\boldsymbol{SS}_{(5,5)}$, $\boldsymbol{HH}_{(7,7)}$]=lu(K$\boldsymbol{K}_{(7,5)}$)

$\boldsymbol{LL}_{(7,5)} =$

1.0000 0 0 0 0
0.2211 1.0000 0 0 0
0.9579 0.7160 1.0000 0 0
0.2105 0.5712 0.2098 1.0000 0
0.8947 0.8411 0.4394 0.9302 1.0000
0.5368 0.8783 0.6794 0.8687 0.4002
0.2316 -0.0240 -0.3301 0.6117 0.0184

$\boldsymbol{HH}_{(7,7)} =$

0 0 0 0 1 0 0
1 0 0 0 0 0 0
0 0 1 0 0 0 0
0 0 0 0 0 0 1
0 1 0 0 0 0 0
0 0 0 0 0 1 0
0 0 0 1 0 0 0

$\boldsymbol{SS}_{(5,5)} =$

95.0000 51.0000 75.0000 53.0000 91.0000
0 33.7263 61.4211 24.2842 48.8842
0 0 -51.8184 -37.1554 -25.1685
0 0 0 32.7666 48.2014
0 0 0 0 -146.3159

such that $\left(HH_{(7,7)}\right)^T * \left(LL_{(7,5)}\right) * \left(SS_{(5,5)}\right) = KK_{(7,5)}$

$S_{(7,5)}$
$= \left(L_{(7,7)}\right)^T * \left(HH_{(7,7)}\right)^T * \left(LL_{(7,5)}\right) * \left(SS_{(5,5)}\right)$
$* \left(H_{(5,5)}\right)$

$$\left.\begin{array}{c}
\sum_{t=1}^{5}\left[\left(\sum_{p=1}^{7}\left\{LL_{(p,t)} * \sum_{i=1}^{7} L_{(i,1)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{5} SS_{(t,j)} * H_{(j,1)}\right)\right] = S_{(1,1)} \\
\sum_{t=1}^{5}\left[\left(\sum_{p=1}^{7}\left\{LL_{(p,t)} * \sum_{i=1}^{7} L_{(i,2)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{5} SS_{(t,j)} * H_{(j,2)}\right)\right] = S_{(2,2)} \\
\vdots \\
\sum_{t=1}^{5}\left[\left(\sum_{p=1}^{7}\left\{LL_{(p,t)} * \sum_{i=1}^{7} L_{(i,q)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{5} SS_{(t,j)} * H_{(j,q)}\right)\right] = S_{(q,q)}
\end{array}\right\} \quad q = min\{7,5\}$$

$$\& \sum_{t=1}^{5}\left[\left(\sum_{p=1}^{7}\left\{LL_{(p,t)} * \sum_{i=1}^{7} L_{(i,w)} * HH_{(p,i)}\right\}\right) * \left(\sum_{j=1}^{5} SS_{(t,j)} * H_{(j,z)}\right)\right] = 0 \ \forall w \neq z$$

To ensure the accuracy of the results, we can program the last relationships above in the MATLAB program and extract the accurate results, and this is what we will do

```
n=7;
m=5;
q=5;
K=[21 45 78 36 69;85 74 96 82 10;91 73 64 31 97;22 11 33 44 55;95 51 75 53 91;51 57 59 53 58;20
30 40 50 90];
[L,S,H]=svd(K);
[LL,SS,HH]=lu(K);
sum4=0;
for t=1:m
 sum1=0;
 for p=1:n
 sum2=0;
 for i=1:n
 sum2=sum2+L(i,q)*HH(p,i);
 end
 sum1=sum1+LL(p,t)*sum2;
 end
 sum3=0;
 for j=1:m
 sum3=sum3+SS(t,j)*H(j,q);
 end
 sum4=sum4+sum1*sum3;
end
sss=sum4
S(q,q)
```

The accuracy of the work became evident when the value of (sss) calculated each time was equal to the corresponding single value S (q, q) calculated by means of (SVD) technique.

5. CONCLUSIONS;-

The relationships obtained above, which we verified through its computer programming on Matlab language, represent a detailed explanation of the nature of the mathematical links between the components of the two analyzes (SVD) and (LU). As it is evident through the overlapping sums between the elements of the matrices, which ultimately equals the single value resulting from the analysis of (SVD), The great flexibility enjoyed by the relationship between these two analyzes. Where we can manipulate the elements that go into the nested sums to produce new matrices with new specific specifications. This flexibility in the relationship between these two technologies is of great use in the field of image processing, compression and encryption within the field of information security.

Of course, this possibility that is characterized by the flexibility of the relationship between these two technologies will double if we combine them with the flexibility of the relationship between (SVD) and (GSVD) technology in which we wrote a previous research, after deducing the relationship between (LU) technology and (GSVD) through research A third we will publish later.
It is quite natural for these algorithms to develop after other research has been written to find relationships between matrices of any two analyzes from all matrix analyzes such as (QR), (CHOL), (ILU), and (GF)... etc.

6. DISCUSSION
The relationships that we extracted above represent a complex analysis of all the components of technologies (SVD) and (LU) and are available for the user to deal with them as they want in the formation of arrays and vectors that serve his goal under investigation, especially after it was found accurate through the Matlab program, which showed that the error is equal to zero.
The broadest field for using the above relationships is the cryptography field in general. The process of encryption the image differs from encryption the text, and for you, because the image encryption bears the error in retrieving the data, either the text encryption does not bear any error, so the effort used in encoding the text is greater than the effort used in encrypting the image in terms of accuracy in work.
The above relationships can be used with the relationships inferred in previous research was to find the relationship between (SVD) analysis and (GSVD) analysis to extract the relationship between (LU) technology and (GSVD).
As our fixed context, we use Matlab to ensure the accuracy of the results.
Work is underway to extract the relationship between any two techniques from matrix analysis techniques, as mentioned in previous research in addition to other techniques.

7. **Future Research:**
May be we will suffice for now with the work projects that we included in the previous research as future work. However, the arena remains open for many other works in the field of information security and other mathematical, physical and astronomical fields.
We ask God Almighty to grant us success during our research work and to guide our steps and guide us to success.

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