A New Fuzzy Technique for Drug Concentration in Blood

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Abstract

The use of integral transforms to obtain a form of solution to differential equations is a very general technique. This powerful methods allows one to turn a complicated problem into a simpler one.

In this paper, we introduce a new integral transform which is called 'SHA-transform' that is used to solve ordinary differential equations. Some important definitions, properties and theorems are discussed as well. The concepts of fuzzy sets are widely used in differential equations field, so in this work, a fuzzy SHA-transform, definitions and established theorems are represented. An equation related with the drug concentration in the plasmas is solved using fuzzy SHA- transform, due to integral transforms accuracy. Drug concentration drives by collecting a blood sample after taking the drug at any time, then measuring the drug's amount in a given volume of the plasma in the sample.

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1. Introduction.

Mathematically, integral transforms converts a function from its original space to another space via integration. The transformed function generally be mapped back to its original one using the inverse transform. In this work, we derived a new integral transform that is called "SHA-transform" with its related theorems.

Fuzzy transforms is a technique belongs to fuzzy approximation models. Fuzzy derivative was first introduced by Chang and Zadeh (1972) [6,11], whereas the concept of fuzzy differential equations was introduced by Kandel and Byatt (1978-1980) [7]. After a while of time, the method of numerical solution was introduced for solving fuzzy differential equations by Abbasbandy and Allahviranloo (2002) [5]. This paper constructs a new fuzzy transform based on "SHA-transform" for solving fuzzy differential equations due to its accurate. The measured drug concentration is demonstrated as an application for this fuzzy procedure. In fact, this drug measurement is generally known as " plasma concentration". An extension for the SHA-transform's kernel is discussed.

2.Main Result

Definition1:: Let $\Im(\delta)$ is function defined for $\delta \geq 0$, let $\mu(\varepsilon)=\varepsilon, \varepsilon \neq 0$ be positive real functions and $\psi(\varepsilon)=i\sqrt[2n]{\varepsilon}+\varepsilon$ be positive complex function , the general integral transform SHA(ε) of $\Im(\delta)$ can be defined by the following formula:

$$(1) \quad SHA\left\{\Im(\delta),\varepsilon\right\}=\mu(\varepsilon)\int_0^\infty \Im(\delta)\, e^{-\psi(\varepsilon)\delta}d\delta = \varepsilon\int_0^\infty \Im(\delta)\, e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\delta \text{ , n} \geq 1 \quad .$$

Definition 2(The Inverse of SHA-Transform): The SHA-transform inverse of $S(\varepsilon)$ denoted by $SHA^{-1}\left[S(\varepsilon)\right]$ is the piecewise continuous function $\Im(\delta)$ on $[0,\infty)$ which satisfies:

$$(2) \qquad SHA\left|\Im(\delta)\right| = S(\varepsilon) \quad .$$

3. SHA-Transform forSome Basic Functions:

Assume that for any function $\Im(\delta)$, the integral in equation (1) exists.SoSHA-transformwill be:

Function $\Im(\delta)=T^{-1}\left\{SHA(\varepsilon)\right\}$	$SHA(\varepsilon)=SHA\left\{\Im(\delta),\varepsilon\right\}$
$\Im(\delta)=1$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon}$, n ≥ 1
$\Im(\delta)=\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2}$, n ≥ 1
$\Im(\delta)=\delta^\alpha$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\Gamma(\alpha+1)\varepsilon}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^{\alpha+1}}$, $\alpha \geq 0$,n ≥ 1.
$\Im(\delta)=\sin\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2+1}$, n ≥ 1
$\Im(\delta)=\cos\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)}{(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2-1}$, n ≥ 1
$\Im(\delta)=\sinh\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2-1}$, n ≥ 1
$\Im(\delta)=\cosh\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2}{(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2-1}$, n ≥ 1
$\Im(\delta)=e^\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon-1}$, n ≥ 1

Following are proofs for some SHA-transform of some functions:

1- If $\Im(\delta)=1$, then $SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon}$, n ≥ 1

$$SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty 1\, e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\delta = \frac{\varepsilon}{-i\sqrt[2n]{\varepsilon}+\varepsilon}\int_0^\infty -e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\,d\delta = \frac{\varepsilon}{-i\sqrt[2n]{\varepsilon}+\varepsilon}\left[e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}\right]_0^\infty = \frac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon} .$$

2- If $\Im(\delta)=\sin\delta$ then

$SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \sin\delta\; e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta$ *integrating by parts* :

$$(3)\qquad \int_0^\infty \sin\delta\; e^{-(\sqrt[2n]{i\,\varepsilon}+\varepsilon)\delta}d\,\delta \;=\left[\sin\delta\frac{-e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}}{i\sqrt[2n]{\varepsilon}+\varepsilon}\right]_0^\infty -\int_0^\infty \cos\delta\;\frac{-e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}}{i\sqrt[2n]{\varepsilon}+\varepsilon}\;d\,\delta$$

Integrating by parts :

$$(4)\qquad \int_0^\infty \cos\delta\;\frac{-e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}}{i\sqrt[2n]{\varepsilon}+\varepsilon}\;d\,\delta\;=\left[\mathrm{con}\delta\frac{-e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}}{i\sqrt[2n]{\varepsilon}+\varepsilon}\right]_0^\infty -\int_0^\infty \text{-}\sin\delta\;\frac{-e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}}{i\sqrt[2n]{\varepsilon}+\varepsilon}\;d\,\delta$$

We compensate (4) *in* (3) $\varepsilon\int_0^\infty \sin\delta\; e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta\;=\dfrac{\varepsilon}{(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2+1}$

then $SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \sin\delta\; e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta\;=\dfrac{\varepsilon}{(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2+1}$

3- If $\Im(\delta)=e^{\delta}$ then $SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{i\sqrt[2n]{\varepsilon}+\varepsilon-1}$, $\mathrm{n}\geq 1$

$$SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \mathrm{e}^{\delta}e^{-i\,(\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta\;=\;\frac{\varepsilon}{e^{\,i\sqrt[2n]{\varepsilon}+\varepsilon}-1}.$$

Theorem1: Let $\Im(\delta)$ is a differentiable function for $\delta\geq 0$ and $\mu(\varepsilon)=\varepsilon,\varepsilon\neq 0$ be real positive function, $\psi(\varepsilon)=i\sqrt[2n]{\varepsilon}+\varepsilon,\varepsilon\neq 0$ be positive complex function then **:**

(I) $SHA\left\{\Im(\delta)',\varepsilon\right\}=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA(\varepsilon)-\varepsilon\;\Im(0)$, $\mathrm{n}\geq 1$,

(Π) $SHA\left\{\Im(\delta)'',\varepsilon\right\}=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2 SHA(\varepsilon)-\varepsilon\;(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\Im(0)-\varepsilon\;\Im(0)'$, $\mathrm{n}\geq 1$,

(IΠ) $SHA\left\{\Im(\delta)^{(n)},\varepsilon\right\}=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^n SHA(\varepsilon)-\varepsilon\;\sum_{k=0}^{n-1}(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^{n-1-k}\;\;\Im(0)^k$, $\mathrm{n}\geq 1$.

Proof :

(I) since $SHA\left\{\Im(\delta)',\varepsilon\right\}=\varepsilon\int_0^\infty \Im(\delta)'e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta$, *integrating by parts* :

$$SHA\left\{\Im(\delta)',\varepsilon\right\}=\varepsilon\left[\Im(\delta)e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}\right]_0^\infty+\int_0^\infty \Im(\delta)e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA(\varepsilon)-\varepsilon\;\Im(0)\text{ , n}\geq 1.$$

(Π) since $SHA\left\{\Im(\delta)'',\varepsilon\right\}=\varepsilon\int_0^\infty \Im(\delta)''e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta$, *integrating by parts* :

$$=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA\left\{\Im(\delta)',\varepsilon\right\}-\varepsilon\Im(0)'=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)^2 SHA(\varepsilon)-\varepsilon\;(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\Im(0)-\varepsilon\;\Im(0)^{\backslash}\text{ , n}\geq 1,$$

(*ΠI*) For this n-th derivative can be prove using mathematical induction.

Theorem2(Convolution): Suppose that $\Im_1(\delta)$ and $\Im_2(\delta)$ are integrable functions with SHA-transforms on [0, +∞),and there are SHA-transforms defined when $\varepsilon \neq 0$, $SHA\left\{\Im_1(\varepsilon)\right\} = \wp$, $SHA\left\{\Im_2(\varepsilon)\right\} = \mathrm{H}$,Then the SHA- transform of their convolution $\Im_1(\varepsilon) * \Im_2(\varepsilon)$ can defined by:

(5) $SHA\left\{\Im_1 * \Im_2\right\}(\varepsilon) = \dfrac{1}{\varepsilon}\wp(\varepsilon)\mathrm{H}(\varepsilon)$

Proof:

$$SHA(\Im_1(\varepsilon) * \Im_2(\varepsilon)) = \varepsilon\int_0^\infty (\Im_1(\varepsilon) * \Im_2(\varepsilon))e^{-(\mathrm{i}\ \sqrt[2n]{\varepsilon}+\varepsilon)\delta} = \varepsilon\int_0^\infty e^{-(\mathrm{i}\ \sqrt[2n]{\varepsilon}+\varepsilon)\delta}\int_0^\infty \Im_1(\delta)\Im_2(\delta-\tau)d\tau$$

$$= \varepsilon\int_0^\infty \Im_1(\delta)\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}\Im_2(\delta-\tau)d\tau = \frac{1}{\varepsilon}SHA_1(\varepsilon)SHA_2(\varepsilon).$$

Next theorem shows the relationship between SHA-transform and Laplace transform.

Theorem3(Duality between Laplace Transform and SHA-Transform): Let $\Im(\delta)$ be a differentiable function if F is Laplace transform of $\Im(\delta)$ and $SHA(\varepsilon)$ is SHA-Transform of $\Im(\delta)$, then:

(6) $SHA(\varepsilon) = \varepsilon F\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)$

Proof:

From Definition 1: $SHA(\varepsilon) = SHA[\Im(\delta);\varepsilon] = \varepsilon\int_0^\infty e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\delta}\Im(\delta)d\delta$, since Laplace transform is denoted by $F(p) = \ell[f(t);p] = \int_0^\infty e^{-pt}f(t)dt$, Then $SHA(\varepsilon) = \varepsilon F\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)$.

Theorem4.(Linearity property):Let $\Im_1(\delta), \Im_2(\delta), ..., \Im_n(\delta)$ are continuous functions have SHA-transforms and $\mu_1, \mu_2, ...\mu_n$ are arbitrary constants ,then linearity property is defined by

$$SHA\left[\mu_1\Im_1(\delta)\right] + \left[\mu_2\Im_2(\delta)\right] + ... + \left[\mu_n\Im_n(\delta)\right] = \mu_1 SHA\left[\Im_1(\delta)\right] + \mu_2 SHA\left[\Im_2(\delta)\right] + ... + \mu_n SHA\left[\Im_n(\delta)\right]$$

Proof:

Let $\Im_1(\delta)$ and $\Im_2(\delta)$ are continuous functions for $\delta \geq 0$:

$$SHA\left[\mu_1\Im_1(\delta) + \mu_2\Im_2(\delta) + ... + \mu_n\Im_n(\delta)\right] = \varepsilon\int_0^\infty \left[\mu_1\Im_1(\delta) + \mu_2\Im_2(\delta) + ... + \mu_n\Im_n(\delta)\right]\ \mathrm{e}^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\delta$$

$$= \varepsilon\int_0^\infty \left[\mu_1\Im_1(\delta)\right]\ \mathrm{e}^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\delta + \varepsilon\int_0^\infty \left[\mu_2\Im_2(\delta)\right]\ \mathrm{e}^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\delta + ... + \varepsilon\int_0^\infty \left[\mu_n\Im_n(\delta)\right]\ \mathrm{e}^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\delta$$

$$= \mu_1 SHA\left[\Im_1(\delta)\right] + \mu_2 SHA\left[\Im_2(\delta)\right] + ... + \mu_n SHA\left[\Im_n(\delta)\right]$$

Illustrative Examples : The following examples usingSHA-transform to solve initial value problems described by ordinary differential equation.

Example 1: Solve $y' + 2y = 0$; $\mathrm{y}(0) = 1$

Applying SHA-transform for both sides of original equation:

$SHA\left[y'\right]+2SHA\left[y\right]=0$

$\Rightarrow\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)SHA\left[y\right]-\varepsilon y\left(0\right)+2SHA\left[y\right]=0$ *,using the initial condition, to obtain* :

$\Rightarrow\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)SHA\left[y\right]+2SHA\left[y\right]-\varepsilon=0\Rightarrow\left(\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)+2\right)SHA\left[y\right]=\varepsilon\Rightarrow SHA\left[y\right]=\dfrac{\varepsilon}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)+2}$.

Applying the inverse of SHA-transform for the last equation: $y=e^{-2\delta}$.

Example 2: Solve $y''+y=0$; $y(0)=0, y'(0)=1$

Applying SHA-transform for both sides of original equation:

$SHA\left[y''\right]+SHA\left[y\right]=0$

$\Rightarrow\left(\sqrt[2n]{i\varepsilon}+\varepsilon\right)^2 SHA\left[y\right]-\varepsilon\left(\sqrt[2n]{i\varepsilon}+\varepsilon\right)y\left(0\right)-\varepsilon y'\left(0\right)+SHA\left[y\right]=0$ *Using the initial conditions* :

$\Rightarrow\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[y\right]+SHA\left[y\right]-\varepsilon=0\Rightarrow\left(\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2+1\right)\left[y\right]=\varepsilon\Rightarrow SHA\left[y\right]=\dfrac{\varepsilon}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2+1}$

Applying inverse of SHA-transform for the last equation, we get: $y=\sin\varepsilon$.

4. Fuzzy SHA-Transform.

In the last decades, fuzzy differential equations have been used in many fields due to their numerous and important applications in a wide range of fields[7,9]. In order to keep pace with the rapid development and progress in the field of fuzzy differential equations, we presented this paper that contains a new method for solving this type of equations[5,6], In this section, we convert the SHA-Transform into a fuzzy SHA-transformand introduce some properties as well as fuzzy theories of the first and second orderthen use these formulas to solve realistic problem which is drug concentration in blood.

Definition 3[4]:A fuzzy number β in parametric form is a pair $\left(\underline{\beta},\overline{\beta}\right)$ of functions $\underline{\beta}(\phi),\overline{\beta}(\phi)$, $0\le\phi\le1$, which satisfy the following requirements:

1. $\underline{\beta}(\phi)$ is a bounded non-decreasing left continuous function in $(0,1]$,and right continuous at 0.
2. $\overline{\beta}(\phi)$ is a bounded non-increasing left continuous function in $(0,1]$,and right continuous at 0.
3. $\underline{\beta}(\phi)\le\overline{\beta}(\phi)$, $0\le\phi\le1$. For arbitrary $\beta=\underline{\beta}(\phi),\overline{\beta}(\phi)$ for $\alpha=\underline{\alpha}(\phi),\overline{\alpha}(\phi)$ and $\varphi>0$ we define addition $\beta\oplus\alpha$,subtraction $\beta\ominus\alpha$ and scalar multiplication by $\varphi>0$ as following

(a) Addition: $\beta\oplus\alpha=\underline{\beta}(\phi)+\underline{\alpha}(\phi),\overline{\beta}(\phi)+\overline{\alpha}(\phi)$.

(b) Subtraction: $\beta\ominus\alpha=\underline{\beta}(\phi)-\overline{\alpha}(\phi),\underline{\alpha}(\phi),\overline{\beta}(\phi)-\underline{\alpha}(\phi)$.

(c) Scalar multiplication: $\varphi\odot\beta=\begin{Bmatrix}\left(\varphi\underline{\beta},\varphi\overline{\beta}\right) & \varphi\ge0\\ \left(\varphi\overline{\beta},\varphi\underline{\beta}\right) & \varphi<0\end{Bmatrix}$.

Definition4.[4] :Let $\beta,\alpha \in \mathrm{E}$(E the set of all fuzzy number) If there exists $\eta \in \mathrm{E}$ such that $\beta+\alpha=\eta$ then η is named Hukuhara difference of β , α and is identify by $\beta \ominus \alpha$.

Note: The sign "$\ominus$" always stands for Hukuhara difference.

Definition 5.[7]: Let $\varphi(\sigma):(a,b) \to \mathrm{E}$(E the set of all fuzzy number) continuous fuzzy- valued function and $\sigma_0 \in (a,b)$,it has been that φ is strongly generalized differential at σ_0 if an aspect exists an element $\varphi^{\backslash}(\sigma_0) \in \mathrm{E}$ such that :

1- For all $\forall\ h>0$ sufficiently small $\exists \varphi(\sigma_0+h) \ominus \varphi(\varpi_0), \exists \varphi(\sigma_0) \ominus \varphi(\sigma_0-h)$ and the

$$limit\ \ is\ \ \varphi^{\backslash}(\sigma_0)=\lim_{h\to 0^+}\frac{\kappa(\sigma_0+h)\ominus\varphi(\sigma_0)}{h}=\lim_{h\to 0^+}\frac{\varphi(\sigma_0)\ominus\varphi(\sigma_0-h)}{h} \ .$$

Or

2- For all $\forall\ h>0$ sufficiently small $\exists \varphi(\sigma_0) \ominus \varphi(\sigma_0+h), \exists \varphi(\sigma_0-h) \ominus \varphi(\sigma_0)$ and the

$$limit\ \ is\ \ \varphi^{\backslash}(\sigma_0)=\lim_{h\to 0^+}\frac{\varphi(\sigma_0)\ominus\kappa(\sigma_0+h)}{-h}=\lim_{h\to 0^+}\frac{\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)}{-h} \ .$$

Or

3- For all $h>0$ sufficiently small $\exists \varphi(\sigma_0+h) \ominus \varphi(\sigma_0), \exists \varphi(\sigma_0-h) \ominus \varphi(\sigma_0)$ and the limitis

$$\varphi^{\backslash}(\sigma_0)=\lim_{h\to 0^+}\frac{\varphi(\sigma_0+h)\ominus\varphi(\sigma_0)}{h}=\lim_{h\to 0^+}\frac{\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)}{-h} \ .$$

Or

4- For all $h>0$ sufficiently small $\exists \varphi(\sigma_0) \ominus \varphi(\sigma_0+h), \exists \varphi(\sigma_0-h) \ominus \varphi(\sigma_0)$ and the limit is

$$\varphi^{\backslash}(\sigma_0)=\lim_{h\to 0^+}\frac{\varphi(\sigma_0)\ominus\varphi(\sigma_0+h)}{-h}=\lim_{h\to 0^+}\frac{\varphi(\sigma_0-h)\ominus\varphi(\sigma_0)}{h} \ .$$

Theorem 5. [5]: Assume that $\varphi: R \to \mathrm{E}$ (E: the set of all fuzzy numbers)be a differentiable function and indicate $\varphi(\sigma)=\left(\overline{\varphi}(\varpi;\phi),\underline{\varphi}(\varpi;\phi)\right)$ for each $\phi \in [0,1]$. Then:

1- If φ is the first form, then $\underline{\varphi}(\sigma;\phi)$ and $\overline{\varphi}(\sigma;\phi)$ are differentiable functions and $\varphi^{\backslash}(\sigma)=\underline{\varphi}(\sigma;\phi),\overline{\varphi}(\sigma;\phi)$.

2- If φ is the second form, then $\underline{\varphi}(\sigma;\phi)$ and $\overline{\varphi}(\sigma;\phi)$ are differentiable functions and $\varphi^{\backslash}(\sigma)=\overline{\varphi}(\sigma;\phi),\underline{\varphi}(\sigma;\phi)$.

Theorem 6. [3,6]: Let $\varphi: R \to \mathrm{E}$ (E: the set of all fuzzy numbers)and is represented by $\left[\underline{\varphi}(\sigma;\phi),\overline{\varphi}(\sigma;\phi)\right]$. For any fixed $\phi \in (0,1]$ assume that $\underline{\varphi}(\sigma;\phi)$ and $\overline{\varphi}(\sigma;\phi)$ are Riemann-integrable functions on [a, b] for every $b \ge a$, and assume there are two positive $\underline{M}_{\phi}$ and $\overline{M}_{\phi}$ such that $\int_a^b \left|\underline{\varphi}(\sigma;\phi)\right| d\sigma \le \underline{M}_{\phi}$ and $\int_a^b \left|\overline{\varphi}(\sigma;\phi)\right| d\sigma \le \overline{M}_{\phi}$ for every $b \ge a$ .Then, $\varphi(\sigma)$ is an improper fuzzy Riemann-integrable on $[a,\infty)$. Furthermore, we have: $\int_a^\infty \varphi(\sigma)\, d\sigma = \left[\int_a^\infty \underline{\varphi}(\sigma;\phi)\, d\sigma, \int_a^\infty \underline{\varphi}(\sigma;\phi)\, d\sigma\right]$.

Definition6: Let $\varphi(\sigma)$ be a continuous fuzzy-valued function Suppose that $\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)d\sigma$ is improper fuzzy Riemann-integrable on $[0,\infty)$, then $\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)d\sigma$ is called SHA -transform and it denoted by:

$$SHA\left[\varphi(\sigma)\right] = SHA(\varepsilon) = \varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)d\sigma$$

$$Sine\ from\ theorem\ 2\ to\ get: \varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\varphi(\sigma)d\sigma = \varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\underline{\varphi}(\sigma;\phi)d\sigma, \varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\overline{\varphi}(\sigma;\phi)d\sigma$$

Using the definition of classic SHA-transform :

$$SHA\left[\underline{\varphi}(\sigma;\phi)\right] = \varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\underline{\varphi}(\sigma;\phi)d\sigma,$$

$$SHA\left[\overline{\varphi}(\sigma;\phi)\right] = \varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\overline{\varphi}(\sigma;\phi)d\sigma$$

So: $SHA\left[\varphi(\sigma;\phi)\right] = SHA\left[\underline{\varphi}(\sigma;\phi)\right], \mathrm{SHA}\left[\overline{\varphi}(\sigma;\phi)\right]$

.

Theorem7. Duality Between Fuzzy Laplace – SHA transforms

If F(p) is fuzzy Laplace transform of $\varphi(\sigma)$ and $SHA(\varepsilon)$ is SHA -transformof $\varphi(\sigma)$ then $SHA(\varepsilon) = \varepsilon F\left(\sqrt[2n]{\varepsilon}+\varepsilon\right)$ **.**

Proof: Let $\varphi(\sigma)\in E$ then: $SHA(\varepsilon) = \left[\varepsilon\int_0^\infty \underline{\varphi}(\sigma;\phi)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\sigma}d\sigma\ ,\ \varepsilon\int_0^\infty \overline{\varphi}(\sigma;\phi)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\sigma}d\sigma\right]$

Since fuzzy Laplace transform denoted by: $F(p) = \left[\int_0^\infty \underline{f}(t;\vartheta)e^{-pt}\,dt\ ,\ \int_0^\infty \overline{f}(t;\vartheta)e^{-pt}\,dt\right]$

$\Rightarrow SHA(\varepsilon) = \left[\varepsilon\int_0^\infty \underline{\varphi}(\sigma;\phi)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\sigma}d\sigma\ ,\ \varepsilon\int_0^\infty \overline{\varphi}(\sigma;\phi)e^{-\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\sigma}d\sigma\right]$ **Thus** $SHA(\varepsilon) = \varepsilon F\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)$

Theorem 8. Let $\varphi_1(\sigma),\varphi_2(\sigma),...,\varphi_n(\sigma)$ be continuous fuzzy- valued functions and suppose that $\eta_1,\eta_2,...,\eta_n$ are constant, then

$$SHA\left[\left(\eta_1 \text{ e } \varphi_1(\sigma)\right)\oplus\left(\eta_2 \text{ e } \varphi_2(\sigma)\right)\oplus...\oplus\left(\eta_2 \text{ e } \varphi_n(\sigma)\right)\right] = \left(\eta_1 \text{ e } SHA\left[\varphi_1(\sigma)\right]\right)\oplus\left(\eta_2 \text{ e } SHA\left[\varphi_2(\sigma)\right]\right)$$
$$\oplus...\oplus\left(\eta_2 \text{ e } SHA\left[\varphi_n(\sigma)\right]\right)$$

proof:

$$\mathcal{S}HA\begin{bmatrix}\left(\eta_1 \mathrm{e}\ \varphi_1(\sigma)\right)\oplus\left(\eta_2 \mathrm{e}\ \varphi_2(\sigma)\right)\\ \oplus\ldots\oplus\left(\eta_2 \mathrm{e}\ \varphi_n(\sigma)\right)\end{bmatrix}=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\left[\left(\eta_1 \mathrm{e}\ \varphi_1(\sigma)\right)\oplus\left(\eta_2 \mathrm{e}\ \varphi_2(\sigma)\right)\oplus\ldots\oplus\left(\eta_2 \mathrm{e}\ \varphi_n(\sigma)\right)\right]d\sigma$$

$$=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\left[\left(\eta_1\underline{\varphi_1}(\sigma)\right)+\left(\eta_2\underline{\varphi_2}(\sigma)\right)+\ldots+\left(\eta_n\underline{\varphi_n}(\sigma)\right)\right],\left[\left(\eta_1\overline{\varphi_1}(\sigma)\right)+\left(\eta_2\overline{\varphi_2}(\sigma)\right)+\ldots+\left(\eta_n\overline{\varphi_n}(\sigma)\right)\right]d\sigma$$

$$=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\begin{bmatrix}\left[\left(\eta_1\underline{\varphi_1}(\sigma)\right),\left(\eta_1\overline{\varphi_1}(\sigma)\right)\right]+\left[\left(\eta_2\underline{\varphi_2}(\sigma)\right),\left(\eta_2\overline{\varphi_2}(\sigma)\right)\right]\\ +\ldots+\left[\left(\eta_n\underline{\varphi_n}(\sigma)\right),\left(\eta_n\overline{\varphi_n}(\sigma)\right)\right]\end{bmatrix}d\sigma$$

$$=\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\left[\left(\eta_1\underline{\varphi_1}(\sigma)\right),\left(\eta_1\overline{\varphi_1}(\sigma)\right)\right]d\sigma+\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\left[\left(\eta_2\underline{\varphi_2}(\sigma)\right),\left(\eta_2\overline{\varphi_2}(\sigma)\right)\right]d\sigma$$

$$+\ldots+\varepsilon\int_0^\infty e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\sigma}\left[\left(\eta_n\underline{\varphi_n}(\sigma)\right),\left(\eta_n\overline{\varphi_n}(\sigma)\right)\right]\mathrm{d}\sigma=\left(\eta_1 \mathrm{e}\ \mathcal{S}HA\left[\varphi_1(\sigma)\right]\right)\oplus\left(\eta_2 \mathrm{e}\ \mathcal{S}HA\left[\varphi_2(\sigma)\right]\right)$$

$$\oplus\ldots\oplus\left(\eta_2 \mathrm{e}\ \mathcal{S}HA\left[\varphi_n(\sigma)\right]\right)$$

Theorem 9. Assume that $\varphi^{\backslash}(\sigma)$ be continuous fuzzy-valued function and $\varphi(\sigma)$ the primitive of $\varphi^{\backslash}(\sigma)$ on $[0,\infty)$, then:

1. $\mathcal{S}HA\left[\varphi^{\backslash}(\sigma)\right]=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\mathcal{S}HA\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi(0)$,where φ is differentiable in the first form
2. $\mathcal{S}HA\left[\varphi^{\backslash}(\sigma)\right]=-\varepsilon\varphi(0)\ominus\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)\mathcal{S}HA\left[\varphi(\sigma)\right]$, where φ is differentiable in the second form

proof: Since $\varphi^{\backslash}(\sigma)$ is continuous fuzzy-valued function then there are two cases as following:

Case 1. If φ is the first form, for any arbitrary $\phi\in[0,1]$,

$$\mathcal{S}HA\left[\varphi^{\backslash}(\sigma)\right]=SHA\left[\underline{\varphi}^{\backslash}(\sigma,\phi)\right],SHA\left[\overline{\varphi}^{\backslash}(\sigma,\phi)\right]$$

From Theorem 1/1:

$$SHA\left[\underline{\varphi}^{\backslash}(\sigma)\right]=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left[\underline{\varphi}(0,\phi)\right]$$

$$SHA\left[\overline{\varphi}^{\backslash}(\sigma)\right]=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left[\overline{\varphi}(0,\phi)\right]$$

By Theorem 2 : $\mathcal{S}HA\left[\varphi^{\backslash}(\sigma)\right]=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\mathcal{S}HA\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi(0)$

Case 2.

If φ is the second form, for any arbitrary $\phi\in[0,1]$,

$$\mathcal{S}HA\left[\varphi(\sigma)\right]=SHA\left[\underline{\varphi}(\sigma,\phi)\right],SHA\left[\overline{\varphi}(\sigma,\phi)\right]$$

From Theorem 1/1:

$$SHA\left[\overline{\varphi}^{\backslash}(\sigma)\right]=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\left[\overline{\varphi}(0,\phi)\right]$$

$$SHA\left[\underline{\varphi}^{\backslash}(\sigma)\right]=(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\left[\underline{\varphi}(0,\phi)\right]$$

By Theorem 2 : $\mathcal{S}HA\left[\varphi^{\backslash}(\sigma)\right]=-\varepsilon(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\varphi(0)\ominus(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)SHA\left[\varphi(\sigma)\right]$

Theorem10. Assume that, $\varphi(\sigma),\varphi^{\backslash}(\sigma)$ are continuous fuzzy-valued functions on $[0,\infty)$, fuzzy derivative of Fuzzy SHA-transform about second order it will be:

1. If $\varphi,\varphi^{\backslash}$ are first form then:

$$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\varepsilon\varphi^{\backslash}(0)$$

2. If φ is first form and $\varphi^{\backslash}$ second form then:

$$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi^{\backslash}(0)$$

3. If φ is second form and $\varphi^{\backslash}$ first form then:

$$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]-\varepsilon\varphi^{\backslash}(0)$$

4. If $\varphi,\varphi^{\backslash}$ are second form then:

$$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)-\varepsilon\varphi^{\backslash}(0)$$

Proof:

1. $\varphi,\varphi^{\backslash}$ are first form and for any arbitrary $\phi\in[0,1]$, then:

$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right],SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right]$ Form Theorem 1/2:

$$SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi^{\backslash}(0,\phi)}$$

$$SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi^{\backslash}(0,\phi)}$$

By Theorem 2: $\mathcal{S}HA\left[\phi^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\varepsilon\varphi^{\backslash}(0)$

2. φ is first form and $\varphi^{\backslash}$ second form, then:

$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right],SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right]$ Form Theorem 1/2:

$$SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi^{\backslash}(0,\phi)}$$

$$SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi^{\backslash}(0,\phi)}$$

By Theorem 2 : $\mathcal{S}HA\left[\phi^{\backslash\backslash}(\sigma)\right]=-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\varphi^{\backslash}(0)$

3. φ is second form and $\varphi^{\backslash}$ first form, then:

$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right],SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right]$ Form Theorem 1/2:

$$SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi^{\backslash}(0,\phi)}$$
$$SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi^{\backslash}(0,\phi)}$$

By Theorem 2 : $\mathcal{S}HA\left[\phi^{\backslash\backslash}(\sigma)\right]=-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)\ominus\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]-\varepsilon\varphi^{\backslash}(0)$

4. $\varphi,\varphi^{\backslash}$ are second form and for any arbitrary $r\in[0,1]$, then:

$$\mathcal{S}HA\left[\varphi^{\backslash\backslash}(\sigma)\right]=SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right],SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma,\phi)\right]$$

Form Theorem 1/2:

$$SHA\left[\underline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\underline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\varphi}(0,\phi)-\varepsilon\underline{\varphi^{\backslash}(0,\phi)}$$
$$SHA\left[\overline{\varphi}^{\backslash\backslash}(\sigma)\right]=\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\overline{\varphi}(\sigma,\phi)\right]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\varphi}(0,\phi)-\varepsilon\overline{\varphi^{\backslash}(0,\phi)}$$

By Theorem 2 : $\mathcal{S}HA\left[\phi^{\backslash\backslash}(\sigma)\right]=\left(-i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2 SHA\left[\varphi(\sigma)\right]\ominus\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\varphi(0)-\varepsilon\varphi^{\backslash}(0)$

Application: The following real word example is illustrated the usage of $\mathcal{S}HA$ -transform.
Example3[15]: For an oral dosage of a drug as tablet or capsule is determined by the amount of drug present in the plasma at a given time. Thus if we let $\eta^{\backslash}(t)$ to be the rate of drug from a oral dosage that is released and $\eta(t)$ is the response of the human body to a unit input of drug which represents a function of the time. The equation will be:

$$\eta^{\backslash}(t)=\frac{1}{t}\eta(t),\quad \eta(0)=[\underline{\eta}(0;\vartheta),\overline{\eta}(\vartheta;0)]$$

There are two cases:

1. If $\eta(t)$ is the first form, then:

fuzzy SHA-transform for both sides of original equation, we get:

$\mathcal{S}HA[\eta^{\backslash}(t)]=\mathcal{S}HA[\eta(t)]$

Last equation becomes:

$$\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)SHA[\underline{\eta}(\mathrm{t};\phi)]-\varepsilon\underline{\eta}(0;\phi)=SHA[\underline{\eta}(\mathrm{t};\phi)],\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)SHA[\overline{\eta}(\mathrm{t};\phi)]-\varepsilon\overline{\eta}(0;\vartheta)=SHA[\overline{\eta}(\mathrm{t};\phi)],$$

So: $SHA[\underline{\eta}(\mathrm{t};\vartheta)]=\dfrac{\varepsilon}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)-1}\underline{\eta}(0;\vartheta),SHA[\overline{\eta}(\mathrm{t};\vartheta)]=\dfrac{\varepsilon}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)-1}\overline{\eta}(0;\vartheta)$

Now we use inverse FSHA-Transform for last equations:

$\underline{\eta}(\mathrm{t};\vartheta)=e^{\sigma}\underline{\eta}(0;\vartheta),\overline{\eta}(\mathrm{t};\vartheta)=e^{\sigma}\overline{\eta}(0;\vartheta)$

2. If $\eta(\sigma)$ is second form, then:

By using FSHA-Transform for both sides of original equation, we get:

$\mathcal{S}HA[\eta^{\backslash}(\sigma)]=\mathcal{S}HA[\eta(\sigma)]$

Last equation becomes:

$$\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)SHA[\overline{\eta}(\mathrm{t};\phi)]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\overline{\eta}(0;\vartheta)=SHA[\overline{\eta}(\mathrm{t};\phi)],$$

$$\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)SHA[\underline{\eta}(\mathrm{t};\phi)]-\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)\underline{\eta}(0;\phi)=SHA[\underline{\eta}(\mathrm{t};\phi)]$$

By solve above equation:

$$SHA[\underline{\eta}(\mathrm{t};\vartheta)]=\frac{\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2-1}\overline{\eta}(0;\vartheta)-\frac{\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2-1}\underline{\eta}(0;\vartheta)$$

$$SHA[\overline{\eta}(\mathrm{t};\vartheta)]=\frac{\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2-1}\underline{\eta}(0;\vartheta)-\frac{\varepsilon\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)}{\left(i\sqrt[2n]{\varepsilon}+\varepsilon\right)^2-1}\underline{\eta}(0;\vartheta)$$

Now we use inverse FSHA-Transform for last equations:

$$\underline{\eta}(0;\vartheta)=\cosh\ \mathrm{t}\overline{\eta}(0;\vartheta)-\sinh\ \mathrm{t}\underline{\eta}(0;\vartheta),\overline{\eta}(0;\vartheta)=\cosh\ \mathrm{t}\underline{\eta}(0;\vartheta)-\sinh\ \mathrm{t}\overline{\eta}(0;\vartheta)$$

5- Kernel of SHA-Transform Expansion:

In this section, we are working on expanding the kernel of SHA -transform to bein the interval $-\infty \le n \le \infty$,

$$SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \Im(\delta)\ e^{-(\mathrm{i}\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta\ \ ,-\infty\le n\le\infty$$

First Case: If $n=0$ **then**

(7) $SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \Im(\delta)\ e^{(-i\sqrt[2n]{\varepsilon}+\varepsilon)\delta}d\,\delta,$ then $SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^\infty \Im(\delta)\ e^{-\varepsilon\delta}d\,\delta$

A special case which is of Laplace multiplied by a constant or Natural transform

Assume that for any function $\Im(\delta)$, the integral in equation (7) exists. So SHA-transform will be:

Function $\Im(\delta)=T^{-1}\left\{SHA(\varepsilon)\right\}$	$SHA(\varepsilon)=SHA\left\{\Im(\delta),\varepsilon\right\}$
$\Im(\delta)=1$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\frac{1}{\varepsilon}$, n=0
$\Im(\delta)=\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\frac{\varepsilon}{(\varepsilon)^2}$, $\mathrm{n}=0$
$\Im(\delta)=\delta^\alpha$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\frac{\Gamma(\alpha+1)\varepsilon}{(\varepsilon)^{\alpha+1}}$, $\alpha\ge 0$,$\mathrm{n}=0.$
$\Im(\delta)=\sin\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\frac{\varepsilon}{(\varepsilon)^2+1}$, n=0
$\Im(\delta)=\cos\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\frac{\varepsilon(\varepsilon)}{(\varepsilon)^2-1}$, $\mathrm{n}=0$

$\Im(\delta)=\sinh\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{(\varepsilon)^2-1}$, n=0
$\Im(\delta)=\cosh\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon(\varepsilon)^2}{(\varepsilon)^2-1}$, $\text{n}=0$
$\Im(\delta)=e^{\delta}$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{\varepsilon-1}$, $\text{n}=0$

Second Case: If $n\le -1$then

(8) $SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^{\infty}\Im(\delta)\, e^{(-i^{-2\sqrt[n]{\varepsilon}}+\varepsilon)\delta}d\,\delta$,then $SHA\left\{\Im(\delta),\varepsilon\right\}=\varepsilon\int_0^{\infty}\Im(\delta)\, e^{-(\frac{1}{i^{2\sqrt[n]{\varepsilon}}}+\varepsilon)\delta}d\,\delta=\varepsilon\int_0^{\infty}\Im(\delta)\, e^{-(\frac{1+i\,\varepsilon^{2\sqrt[n]{\varepsilon}}}{i^{2\sqrt[n]{\varepsilon}}})\delta}d\,\delta$

Assume that for any function $\Im(\delta)$, the integral in equation (8) exists. SoSHA-transform will be:

Function $\Im(\delta)=T^{-1}\left\{SHA(\varepsilon)\right\}$	$SHA(\varepsilon)=SHA\left\{\Im(\delta),\varepsilon\right\}$
$\Im(\delta)=1$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon i^{2\sqrt[n]{\varepsilon}}}{1+\varepsilon i^{2\sqrt[n]{\varepsilon}}}$, $\text{n}\le -1$
$\Im(\delta)=\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{\left(\dfrac{1+i^{2\sqrt[n]{\varepsilon}}+\varepsilon}{i^{2\sqrt[n]{\varepsilon}}}\right)^2}$, $\text{n}\le -1$
$\Im(\delta)=\delta^{\alpha}$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\Gamma\left(\alpha+1\right)\varepsilon}{\left(\dfrac{1+i\,\varepsilon^{2\sqrt[n]{\varepsilon}}}{i^{2\sqrt[n]{\varepsilon}}}\right)^{\alpha+1}}$, $\alpha\ge 0$,$\text{n}\le -1$.
$\Im(\delta)=\sin\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{\left(\dfrac{1+i\,\varepsilon^{2\sqrt[n]{\varepsilon}}}{i^{2\sqrt[n]{\varepsilon}}}\right)^2+1}$, $\text{n}\le -1$
$\Im(\delta)=\cos\delta$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon\left[\dfrac{1+i\,\varepsilon^{2\sqrt[n]{\varepsilon}}}{i^{2\sqrt[n]{\varepsilon}}}\right]}{\left(\dfrac{1+i\,\varepsilon^{2\sqrt[n]{\varepsilon}}}{i^{2\sqrt[n]{\varepsilon}}}\right)^2+1}$, $\text{n}\le -1$
$\Im(\delta)=e^{\delta}$	$SHA\left\{\Im(\delta),\varepsilon\right\}=\dfrac{\varepsilon}{\left[\dfrac{1+i\,\varepsilon^{2\sqrt[n]{\varepsilon}}}{i^{2\sqrt[n]{\varepsilon}}}\right]-1}$, $\text{n}\le -1$

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