Modeling of Three-Phase Written-Pole Motor and Performance Predetermination using Numerical Methods

Manjunath B. Ranadev¹, V. R. Sheelavant², R. L. Chakrasali³

¹ Department of Electrical and Electronics Engineering, K.L.E. Institute of Technology, Hubballi, 580 027, Karnataka, India.
² Department of Electrical and Electronics Engineering, SDM College of Engineering and Technology, Dharwad, 580 002, Karnataka, India.
³ Department of Electrical and Electronics Engineering, SDM College of Engineering and Technology, Dharwad, 580 002, Karnataka, India.

Abstract
Modeling has attracted increasing attention from researchers. A physical system transformation to a mathematical base is important to analyse the behaviour of the system. The solution of various engineering problems depends on appropriate mathematical models. Differential equations are the basic tool for the mathematical modeling of electric machines. A model that describes the machine performance is used to analyse the dynamic behaviour of a 3-phase written-pole motor.

Mathematical modeling of a 3-phase written-pole motor will help us to do the numerical analysis. It provides insight and useful guidance for originating the applications. A method for studying the behaviour of a 3-phase written-pole motor under steady state condition is described. The numerical tools such as Gauss-Seidel method and Successive Over-Relaxation method are used. This study forms the foundation for further research and development.

Keywords: - flux, linkage, magnetizing, inductance, dq winding, iterative.

I INTRODUCTION
Precise Power Corporation, Florida has evolved into the design of a 3-phase written-pole motor. The Electric Power Research Institute along with National Rural Electric Cooperative Association recognized the potential significant benefit. The current drawn by the written-pole motor during starting can be reduced appreciably by controlling the magnetic field. Also these motors are more efficient than those of conventional induction motors. The written-pole motors are available with the ratings up to 100hp meeting the starting current limitations of utility system. The ductile iron or aluminium castings are used to fabricate motor frame. The stator laminations composed of high grade steel laminations and suitable grade copper wire is used for windings. The rotor is a combination of hysteresis, induction and permanent magnet principle. The rotor comprise
of a steel shaft positioned inside lamination stack. It contains a rotor cage made up of high resistance carbon steel rotor bars. The rotor laminations are composed of the same low loss steel appearing in the stator laminations. The 3-phase written-pole motor with inner rotor construction is shown in Fig.1[1].

![Fig.1 3-phase written-pole motor with inner rotor construction](Curtsy Precise Power Corporation, Palmetto, FL, U.S.A)

Written pole motor exerts three modes of operation depending on the rotational speed. Large amount of hysteresis and induction torque is produced during start mode. During transition mode the excitation winding begins to interact with the rotor magnetic geometry. The excitation winding creates magnetic pole pattern which revolves at exact electromagnetic synchronization that with the field produced by the stator winding. In this mode motor produces synchronous torque and becomes electrically synchronous, even though rotor has not reached true synchronous speed. Upon reaching its rated synchronous speed written-pole motor enters run mode. The excitation winding operation is not required in this mode, hence it is turned off. The motor starts operating as permanent magnet synchronous motor [2,3].

The numerical methods are classified as direct and iterative. In direct method solution to simultaneous linear equations is obtained after performing certain fixed computation. Iterative or indirect method provides a technique to obtain a refined estimate by predicting an approximate value using a systematic method. The solution to a system of equations will exist if the sum of modulus of the remaining coefficients in each equation is less than absolute value of largest coefficient. The Gauss-Seidel method is a modification of Gauss-Jacobi method. At each stage of iteration in Gauss-Seidel method the current values of the unknowns are used in proceeding to the next stage of iteration. This method is more rapid in convergence than that of Gauss-Jacobi method. In this method the rate of convergence is nearly twice compared to that of Gauss-Jacobi method. Successive Over-Relaxation method (SOR) is a revision of Gauss-Seidel method and is developed to enhance convergence.

II DESIGN CONSIDERATIONS

The mutual inductance between the rotor and stator winding is position dependent. Hence, the rotor and the stator flux linkages depend on the rotor angle $\theta_m$ and in turn depend on instantaneous rotor position. The dq-axis analysis based on physical approach to mathematical transformation is
discussed below. Here, $N_s$ represent stator and rotor number of turns per phase. Two orthogonal windings are essential for dynamic analysis and control. Due to orthogonal orientation dq-windings are mutually decoupled magnetically. At any instant of time, two orthogonal d and q windings produce the air gap mmf distribution similar to that of three phase-windings. Each of the stator d and q windings have sinusoidal distribution with $\sqrt{3/2}N_s$ turns. Here, $R_s$ represents equivalent winding resistance and $L_{ls}$ represents leakage inductance. The rotor dq-axis is chosen identical to that of stator. The rotor dq-windings have $\sqrt{3/2}N_s$ turns. Here $R_r$ and $L_{lr}$ represents equivalent resistance and leakage inductance of the stator windings respectively. The magnetizing inductance is $L_m$, because of same magnetic path for flux lines and same number of turns. Since the magnetizing flux penetrates through the air gap, mutual inductance between d-axis windings is $L_m$. Also the mutual inductance between q-axis windings is $L_m$. Due to orthogonal orientation, $L_m$ between any d and q-axis winding is zero. Here, $p$ denotes derivative operator and $P$ means total number of poles [4].

### III Theory Of Dynamic Modelling

The following assumptions are made in three-phase written-pole motor dynamic model. The stator consists of sinusoidal distributed symmetrical three phase windings. Skin effect and constant losses are neglected. The iron used is having infinite permeability. The motor operates in the linear region of the B-H characteristics of the stator and the rotor materials. The motor parameters are function of rotor speed. Hence, the differential equation coefficients are time variant except when the rotor is stalled.

a) Start mode: The equivalent dq-windings of simplified 3-φ written-pole motor are shown in Fig 2 [5].

![Fig.2 Equivalent dq-model of 3-phase written-pole motor during start mode.](image)

The expression for the stator winding flux linkage can be written as the sum of flux linkages because of current in the same winding, magnet source and mutual flux linkage due to current in the other winding. The stator d and q axis flux linkage expressions are written as:

\[
\phi_{sd} = L_s.i_{sd} + L_m.i_{rd} + \phi_m.\cos\theta_{da} \\
\phi_{sq} = L_s.i_{sq} + L_m.i_{rq} + \phi_m.\sin\theta_{da}
\]
Where, \( \phi_m \) is the stator d winding flux linkage caused by the rotor magnet and \( L_s = L_{ds} + L_{m} \). Similarly, flux linkage expressions for the rotor winding are written as:

\[
\phi_{rd} = L_{r}i_{rd} + L_{m}i_{sd} \tag{3}
\]

\[
\phi_{rq} = L_{r}i_{rq} + L_{m}i_{sq} \tag{4}
\]

Where, \( L_r = L_{dr} + L_{m} \). The voltage supplied to the stator should balance the stator resistance voltage drop and to induce the emf required to create the stator flux linkage. Consider a pair of orthogonal \( \alpha \beta \)-winding fixed with the stator. Here \( \alpha \)-axis and stator \( a \)-axis both are aligned. The \( \alpha \beta \) winding voltages are given by

\[
V_{sa} = R_s i_{sa} + p\phi_{sa} \tag{5}
\]

\[
V_{sb} = R_s i_{sb} + p\phi_{sb} \tag{6}
\]

The voltage space vectors of \( \alpha \)-axis and \( d \)-axis are correlated as,

\[
V_{(s, \alpha \beta)} = V_{(s, dq)} \cdot e^{p\theta_{da}} \tag{7}
\]

Where, \( \theta_{da} \) is the angle made by dq-winding with reference to stator \( a \)-axis. Separating real and imaginary component of equation 7 [6].

\[
V_{sd} = R_s i_{sd} - \omega_d \cdot \phi_{sq} + p\phi_{sd} \tag{8}
\]

\[
V_{sq} = R_s i_{sq} + \omega_d \cdot \phi_{sd} + p\phi_{sq} \tag{9}
\]

Where, \( p\theta_{da} = \omega_d \), which represents the instantaneous dq-winding speed in air gap. An analysis of rotor is carried out analogous to the stator. Here \( \alpha \)-axis is aligning with the rotor A-axis. Here \( \theta_{dA} \) is the angle made by dq-winding with reference to rotor A-axis. The rotor voltage equations are

\[
V_{rd} = R_r i_{rd} - \omega_{dA} \cdot \phi_{rq} + p\phi_{rd} \tag{10}
\]

\[
V_{rq} = R_r i_{rq} + \omega_{dA} \cdot \phi_{rd} + p\phi_{rq} \tag{11}
\]

Where, \( p\theta_{dA} = \omega_{dA} \) which represents the instantaneous dq-winding set speed with respect to the rotor A-axis.

\[
\omega_{dA} = \omega_d - \omega_m \tag{12}
\]

Here \( \omega_m \) is having relationship with the actual rotor speed by the pole pairs. The choice of \( \omega_d = \omega_{synchronous} \) (\( \omega_{dA} = \omega_{sAp} \)) results in the hypothetical dq winding speed same as air gap field distribution.

The stator voltage equations in terms of inductances are written as:

\[
V_{sd} = R_s i_{sd} + L_{s} p i_{sd} - \omega_d L_{s} i_{sq} + L_{m} p i_{sd} - \omega_d L_{m} i_{sq} - \omega_m i_{sd} \cdot \sin \theta_{dA} \tag{13}
\]

\[
V_{sq} = \omega_d L_{s} i_{sd} + R_s i_{sq} + L_{s} p i_{sq} + \omega_d L_{m} i_{rd} + L_{m} p i_{sq} + \omega_{dA} \cdot \phi_m \cdot \cos \theta_{dA} \tag{14}
\]

Similarly the rotor voltage equations are written as

\[
V_{rd} = L_{m} p i_{sd} - \omega_{dA} L_{m} i_{sq} + R_r i_{rd} + L_{r} p i_{rd} - \omega_{dA} L_{r} i_{iq} \tag{15}
\]

\[
V_{rq} = \omega_{dA} L_{m} i_{sd} + L_{m} p i_{sq} + \omega_{dA} L_{r} i_{rd} + R_r i_{rq} + L_{r} p i_{rq} \tag{16}
\]

As rotor winding is shorted on both the ends, \( V_{rd} = 0 \) and \( V_{rq} = 0 \). The instantaneous torque is obtained by adding up the torques acting on the rotor \( d \) and the \( q \)-axis windings.

\[
T_{em} = (P/2)(\phi_{rq} \cdot i_{rd} - \phi_{rd} \cdot i_{rq}) \tag{17}
\]

b) Transition mode: The exciter coil is activated with alternating current at line frequency, when the rotor attains 80% of synchronous speed. The rotor semi permeable surface is magnetized by the excitation pole. This enables the rotor to develop synchronous torque and attains synchronous speed.
The temporary poles cause the motor slip to become zero. Thus no current will be induced in the rotor bars. Hence flux linkage expressions can be written as:

\[
\begin{align*}
\phi_{sd} &= L_s i_{sd} + L_m i_f \\
\phi_{sq} &= L_s i_{sq} \\
\phi_f &= L_f i_f + L_m i_{sd}
\end{align*}
\]

The voltage equations may be written as:

\[
\begin{align*}
V_{sd} &= R_s i_{sd} + L_s i_{sd} - \omega_d L_s i_{sq} + L_m i_f \\
V_{sq} &= \omega_d L_s i_{sd} + R_s i_{sq} + L_s i_{sq} + \omega_d L_m i_f \\
V_f &= R_f i_f + L_f i_f + L_m i_{sd}
\end{align*}
\]

c) Run mode: The excitation coil is disconnected after inducing the required number of poles. The motor will continue to run as a permanent magnet synchronous motor. Here \( L \) represents the synchronous inductance which is the effective inductance under balanced three phase conditions. It is made up of self inductance and contributions from other two phase currents [7]. The flux linkage expressions can be written as:

\[
\begin{align*}
\phi_{sd} &= L_s i_{sd} + \phi_m \\
\phi_{sq} &= L_s i_{sq} \\
\phi_f &= L_f i_f + L_m i_{sd}
\end{align*}
\]

The flux linkage of the permanent magnet can be obtained by measuring the no-load line-to-line rms voltage \( V_{nl} \) of the motor while rotating at a constant speed of \( \omega_d \).

\[
\phi_m = \sqrt{(2/3)} \cdot (V_{nl} / \omega_d)
\]

In run mode \( d \)-axis is aligned with the rotor magnetic axis, recognizing that the motor becomes electrically synchronous. The instantaneous voltage equations may be written as:

\[
\begin{align*}
V_{sd} &= R_s i_{sd} + L_s i_{sd} - \omega_d L_s i_{sq} \\
V_{sq} &= \omega_d L_s i_{sd} + R_s i_{sq} + L_s i_{sq} + \omega_d \phi_m \\
V_f &= R_f i_f + L_f i_f + L_m i_{sd}
\end{align*}
\]

The instantaneous real power input and power output in terms of \( dq \) variables are given in equation 29 and equation 30.

\[
\begin{align*}
P_{in} &= (3/2) \cdot (V_{sd} i_{sd} + V_{sq} i_{sq}) \\
P_{out} &= (3/2) \cdot [ - \omega_d \phi_q \cdot i_{sd} + \omega_d \phi_d \cdot i_{sq} ]
\end{align*}
\]

For some applications, it is useful to define voltage vector \( V_s \) and current vector \( I_s \) whose magnitudes are

\[
\begin{align*}
V_s &= \sqrt{V_{sd}^2 + V_{sq}^2} \\
I_s &= \sqrt{I_{sd}^2 + I_{sq}^2}
\end{align*}
\]

Assuming the current vector \( I_s \) is 0 degrees ahead of the \( q \)-axis. Then the relation between the stator current magnitude \( I_s \), \( I_{sd} \) and \( I_{sq} \) are given in equation (33) and (34).

\[
\begin{align*}
I_{sd} &= -I_s \sin \theta \\
I_{sq} &= I_s \cos \theta
\end{align*}
\]

The produced torque, which is power divided by mechanical speed can expressed in terms of \( \theta \) as

\[
T_{em} = (3/4) \cdot (\phi_m I_s \cos \theta)
\]
IV NUMERICAL TOOLS

Iterative or approximate methods provide an alternative to the elimination methods. These techniques are designed to derive the roots of a single equation. These approaches comprise of guessing a value and then used in a systematic procedure to obtain a refined estimate of the root.

a) Gauss-Seidel method: This is the modification of Gauss-Jacobi method. This method is applicable to any convergence matrix and it is the most generally used iterative method. Consider the system of linear equations.

\[ \begin{align*}
    a_1x + b_1y + c_1z &= d_1 \\
    a_2x + b_2y + c_2z &= d_2 \\
    a_3x + b_3y + c_3z &= d_3 
\end{align*} \]  

The above system of equations can be written as

\[ \begin{align*}
    x &= \frac{1}{a_1}(d_1 - b_1y - c_1z) \\
    y &= \frac{1}{b_2}(d_2 - a_2x - c_2z) \\
    z &= \frac{1}{c_3}(d_3 - a_3x - b_3y) 
\end{align*} \]  

(37)

Starting with the initial approximation \( x_0, y_0, z_0 \) in the equation 37, initially \( y_0 \) and \( z_0 \) is substituted in first expression right hand side and the result is denoted by \( x_1 \). In second expression \( x_1 \) and \( z_0 \) is used to obtain the result and is denoted by \( y_1 \). Similarly in the third expression \( x_1 \) and \( y_1 \) are used and the result is denoted by \( z_1 \) and so on. This process is extended till the desired precision is obtained. Since the current values of the unknowns at each stage of iteration are used in proceedings to the next stage of iteration, this method is more rapid in convergence than Gauss-Jacobi method. The Gauss-Seidel iteration method converges only for special systems of equations. In general, the round of errors will be small in iteration methods. Moreover, these are self correcting methods i.e. any error generated in computation will be corrected in the subsequent iteration [8].

b) Successive Over-Relaxation method: It is an iterative method. The basic reason for using relaxation method is to increase the speed of iteration by reducing the largest residual to almost zero. Consider the system of linear equations given in equation 36. The residuals \( r_1, r_2, r_3 \) are defined by the relations

\[ \begin{align*}
    r_1 &= d_1 - a_1x - b_1y - c_1z \\
    r_2 &= d_2 - a_2x - b_2y - c_2z \\
    r_3 &= d_3 - a_3x - b_3y - c_3z 
\end{align*} \]  

(38)

Initially assuming \( x=y=z=0 \), the initial residuals are calculated. Then these residuals are reduced step by step incrementing the variables. The values of \( x, y, z \) for the residuals \( r_1=r_2=r_3 =0 \) are the exact values. Otherwise the residuals are liquidated to smaller and finally negligible to get better approximate values of \( x, y, z \).

At each step, the numerically largest residual is reduced almost to zero. To reduce a particular residual, the value of the corresponding variable is changed. That is to reduce \( r_2 \) by \( \alpha \), \( y \) should be increased by \( (\alpha/b_2) \). When all the residuals have been reduced to almost zero, then increments in \( x, y \) and \( z \) are added separately to give the desired solution. After substituting these \( x, y, z \) values in equation 38, the residuals become negligible. This method can be applied successfully only if the diagonal elements of the coefficient matrix dominate the other coefficients in the corresponding row and with strict inequality for minimum one row.
V SIMULATION RESULTS

A typical written pole motor is used to simulate the steady state behaviour with the following motor specifications. Power: 2.2 kilo Watts / 3HP, Voltage: 415 Volts, Frequency: 50 cycles per second, No. of Phases: 3, Rated Full-Load Current: 4.6 A, No. of Poles: 4, Rated Full-Load Speed: 1400 revolutions per minute. Per-Phase circuit parameters are: \( R_s = 9.3 \, \Omega \), \( R_r = 13.1 \, \Omega \), \( X_s = X_r = 13.94 \, \Omega \) (at 50 cycles per second), \( X_m = 371.13 \, \Omega \) (at 50 cycles per second), and corresponding \( L_s = L_r = 1.2244 \, \text{H} \), \( L_m = 1.18 \, \text{H} \), synchronous speed \( (\omega_s) = 157 \, \text{rad/sec}. \)

The following curves are drawn from the results obtained using Gauss-Seidel method and SOR method.

**Fig 3.** Slip in % versus Stator d-axis flux linkages in weber-turns.

It is depicted from figure 3 that stator d-axis flux linkage \( (\phi_{sd}) \) decreases with increase in slip. Because as the slip increases \( i_{sd} \) decreases. Hence, as slip increases \( \phi_{sd} \) decreases. The value of \( \phi_{sd} \) obtained at 6.26% slip is -1.0985 Wb-turns using Gauss-Seidel method and it is -1.0763 Wb-turns using SOR method.

**Fig 4.** Slip in % versus Stator q-axis flux linkages in weber-turns.

Similarly as slip increases \( i_{sq} \) decreases and hence Stator q-axis flux linkage \( (\phi_{sq}) \) decreases with slip as shown in figure 4. The value of \( \phi_{sq} \) obtained at 6.26% slip is -1.4332 Wb-turns using Gauss-Seidel method and it is -1.4190 Wb-turns using SOR method.
As shown in figure 5, torque increases with increase in slip. It is because as slip increases stator current increases and $\theta$ decreases. This is given in equation 35. The value of torque obtained at 6.26% slip is 22.40 N-m using Gauss-Seidel method. The value of torque at 6.26% slip using SOR method is 22.18 N-m.

The efficiency obtained at 6.26% slip is 85.96% and 85.95% using both Gauss-Seidel method and Successive Over-Relaxation method respectively. The efficiency obtained from Gauss-Seidel and Successive Over-Relaxation methods are comparable and are within the acceptable range as shown in figure 6.
The power factor obtained at 6.26% slip is 0.85 using Gauss-Seidel and Relaxation method. The obtained value of power factor from both Gauss-Seidel and relaxation method are very close. The power factor variation with slip is shown in figure 7. The machine exhibits good power factor at different load conditions.

The speed in rpm versus torque in N-m characteristics is obtained using Gauss-Seidel method resembles ideal characteristics of written pole motor [9]. The Speed-Torque characteristic of a 3-phase written pole motor is shown in figure 8. The torque obtained using Gauss-Seidel method at 1350 rpm (0.1 slip) is 24.33 N-m.

**VI CONCLUSION**

This paper discusses to reassert the performance of written pole motor through mathematical modeling. The dynamic model is used to analyze the performance of written pole motor in start, transition and run mode using dq-model. It is evident that the use of d-q model reduces the computations. The Gauss-Seidel method and Successive Over-Relaxation methods are used for the performance evaluation of written pole motor and the results are compared. The comparison of the results indicates the validity of the methods used as yet another approach. The written pole motor
exhibits high efficiency and good power factor at different loading conditions. The motor develops almost constant and high torque during run condition. Thus, the written pole motor can greatly replace the existing industrial drives with its features of superior performance and ride through advantages.

REFERENCES:


